Symmetry, Fourier series, and jumps

Recall that a function $y = f(x)$ is called *even* if its graph is symmetric with respect to the y-axis; in terms of a formula, this condition says $f(-x) = f(x)$. Even powers of x, like x^4, are even functions. If f is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$.

A function $y = f(x)$ is called *odd* if its graph is anti-symmetric: $f(-x) = -f(x)$. Odd powers of x, like x^5, are odd functions. If f is an odd function, then $\int_{-a}^{a} f(x)\,dx = 0$.

A function like $y = e^x$ is neither even nor odd: its graph has no particular symmetry.

Suppose we start with a function f such that $f(x) = 1$ when $0 < x < \pi$. This function can be extended in different ways to the interval $(-\pi, \pi)$.

1. Extend f to be an even function f_1 on the interval $(-\pi, \pi)$. Draw its graph and find its Fourier series.

2. Extend f to be an odd function f_2 on the interval $(-\pi, \pi)$. Draw its graph and find its Fourier series.

   Notice that the graph of f_2 has a jump (a discontinuity).

3. Extend f to be a non-symmetric function f_3 by setting $f_3(x) = 0$ for $-\pi < x < 0$. Draw its graph and find its Fourier series.

   Notice that the graph of f_3 has a jump (a discontinuity).

4. Suppose you plug $x = 0$ into the three Fourier series that you found. What values do you get, and what do these values have to do with the graphs of the functions f_1, f_2, and f_3?

Hint: think before you calculate. You should not have to compute many integrals.