# Complex Variables

**Instructions** Please write your solutions on your own paper.

These problems should be treated as essay questions. You should explain your reasoning in complete sentences.

1. State the following:
   a) Liouville's theorem (about entire functions); and
   b) Gauss's mean-value theorem (about functions analytic in a disk).

2. Evaluate the line integral $\int_C (z - \overline{z})\,dz$ when $C$ is the circle of radius 1 centered at 0 (oriented in the usual counterclockwise direction). [Caution: The answer is not zero!]

3. Determine the values of the complex number $z$ for which the infinite series $\sum_{n=1}^{\infty} e^{nz}$ is convergent.

4. Evaluate $\int_C \frac{329z + 3\cos(2z)}{z^9}\,dz$ when $C$ is the circle of radius 1 centered at 0 (oriented in the usual counterclockwise direction). [Caution: The answer is not zero!]

5. Determine the radius of convergence of the power series
$$\sum_{n=1}^{\infty} \left( \frac{3 + 2i}{9^n + i^n} \right) z^n.$$

6. When $C$ is a path in the complex plane from the point $-1$ to the point 1, the value of the line integral
$$\int_C \frac{2}{z^2 + 1}\,dz$$
depends on the choice of $C$. If $C$ is the straight line segment along the real axis from $-1$ to 1, then the value of this integral is $\pi$. What are the other possible values for this integral (for other choices of the path $C$ joining $-1$ to 1)?

## Extra credit

Show that if $f$ is an entire function such that $|f(z)| \le 1 + |z|$ for every complex number $z$, then $f$ is a polynomial of degree at most 1.