Math 409-502

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Reminder

Second examination is Monday, November 1.

The exam will have a similar format to the format of the first exam.

The exam covers material through section 13.4.

Continuity and boundedness

A continuous function on an interval need not be bounded.
Examples: $1/x$ on the bounded interval $(0, 1)$;
x^2 on the closed interval $[0, \infty)$.

Theorem: Every continuous function f on a *compact* interval $[a, b]$ is bounded.

Proof (different from the proof in the book):
Use from Exercise 11.1/5 that

$$(*) \text{ every continuous function is } \textit{locally} \text{ bounded.}$$

Let $S = \{\, c : f \text{ is bounded on the interval } [a, c] \,\}$. By $(*)$, S is not empty. Let $d = \sup S$. By $(*)$, $d \in S$. If $d < b$, then by $(*)$ some points to the right of d are in S, which contradicts that d is an upper bound for S. Therefore $d = b$, and we are done.

Continuity and extreme values

A continuous function on an interval need not attain a maximum value.
Examples: x^2 on the bounded interval $(0, 1)$;
$\arctan(x)$ on the closed interval $[0, \infty)$.

Theorem: Every continuous function f on a *compact* interval attains a maximum value (and also attains a minimum value).

Proof by contradiction (different from the proof in the book):
By the previous theorem, f is bounded. Let $M = \sup f(x)$.
Suppose the supremum is not attained.
Then $M - f(x) > 0$ for all x, so $\frac{1}{M-f(x)}$ is continuous.
By the previous theorem, this new function has an upper bound, say N. Solve $\frac{1}{M-f(x)} \leq N$ to get $f(x) \leq M - \frac{1}{N}$, contradicting that $M = \sup f(x)$.

Homework

- Read sections 13.3 and 13.4, pages 187–190.
- In preparation for the examination, make a list of the main definitions, concepts, and theorems from sections 7.5 through 13.4.