Exam results

- Grading scheme: 40 point baseline plus 10 points per problem
- Median $\approx$ mean $\approx$ 77
- One score over 100

Etymology and spelling

Latin *os* = mouth, face

- *oscillate* = to move back and forth, hence to behave like the graph of $\sin(x)$
- *osculate* = to kiss, hence to behave like the graphs of $\sin(x)$ and x

Vocabulary today

- neighborhood
- interior point
- boundary point
- limit point
- isolated point
- open set
- closed set

Intervals in $\mathbb{R}$

- $[1, 4]$ is a *closed* interval (the endpoints are included)
- $(2, 5)$ is an *open* interval (the endpoints are not included)
- $[3, 7) = \{x : 3 \leq x < 7\}$ (neither open nor closed)

Neighborhood, interior point

A point x of a set E is an *interior point* if E contains an open interval around x.
Example. If $E = [1, 3]$, then x is an interior point of E precisely when $1 < x < 3$. But the endpoint 1 is not an interior point, because E contains no open interval that contains 1.
Example. If $E = (2, 5)$, then the interior points are all the points of E.
Example. $E = \mathbb{Q} \subset \mathbb{R}$. This set E has no interior points at all (with respect to the universe $\mathbb{R}$).
The letter E comes from French *ensemble*.
A set E is a *neighborhood* of a point x precisely when x is an interior point of the set E.

A set is *open* if all of its points are interior points.
Example. $E = \mathbb{R} \setminus \mathbb{Z}$ or $\mathbb{R} - \mathbb{Z}$ or $\{x \in \mathbb{R} : x \notin \mathbb{Z}\}$ is an open set: it contains an open interval around each of its points.

The *interior* of a set is the set of all points that are interior points of the set.
Example. If $E = [2, 5]$, then the interior of E is $(2, 5)$. Notation: E°
Example. If $E = \mathbb{Z}$, then $E^\circ = \varnothing$.

Other types of points

A point x (not necessarily a point of E) is a *boundary point* if every neighborhood of x intersects both E and the complement of E.

Example. $E = [2, 5)$ has boundary points 2 and 5.
Example. $E = \mathbb{Q}$. Every real number is a boundary point because every open interval intersects both E and the complement of E.
Example. If $E = \mathbb{R}$, then E has no boundary points.

A point x of a set E is *isolated* if there is some neighborhood of x that contains no other point of E.
Example. $E = \mathbb{Z}$. All the points of E are isolated.

A point x (not necessarily in E) is a *limit point* or *accumulation point* of E if every neighborhood of x contains at least one point of E other than x itself.
Example. $E = [2, 5)$. The limit points are all the points of E and also the point 5.

Example

$E = \mathbb{Z}$. All the points of E are isolated, so these points are boundary points that are not limit points.

Exercise

For each of the following sets, identify the interior points, the boundary points, the isolated points, and the limit points.

- $\{1/2, 1/3, 1/4, \ldots\} = \{1/n : n \in \mathbb{N}, n \geq 2\}$
- $\{0\} \cup \{1/2, 1/3, 1/4, \ldots\}$
- $\mathbb{R} \setminus \mathbb{Z}$
- $\mathbb{R} \setminus \mathbb{Q}$
- $\{x \in \mathbb{R} : x^2 < 2\}$
- $\{x \in \mathbb{Q} : x^2 < 2\}$