

## Some properties a sequence $\{x_n\}_{n=1}^{\infty}$ might have

- ▶ increasing:  $\forall n (x_n \leq x_{n+1})$
- ▶ decreasing:  $\forall n (x_{n+1} \leq x_n)$
- ▶ monotonic (or monotone): either increasing or decreasing
- ▶ bounded:  $\exists B \forall n (|x_n| \leq B)$
- ▶ convergent to limit  $L$ :  $\forall \varepsilon > 0 \exists M_\varepsilon \forall n \geq M_\varepsilon (|x_n - L| < \varepsilon)$   
Verbal shorthand: “for every neighborhood of  $L$ , the sequence is eventually in the neighborhood.”
- ▶ Cauchy:  $\forall \varepsilon > 0 \exists M \forall n \geq M \forall m \geq M (|x_n - x_m| < \varepsilon)$

Which properties do these sequences satisfy?

(a)  $x_n = 1/n$

(b)  $x_n = \cos(\pi n)$

(c)  $x_n = 2^n$

(d)  $x_n = 2^{1/n}$

(e)  $x_n = \cos(n)$

Are these sequences increasing? decreasing? monotonic?  
bounded? convergent? Cauchy?

## Assignment due next class

- ▶ Write solutions to Exercises 2.1.14 and 2.1.19 (both easy).
- ▶ Read subsection 2.2.1 in the textbook (about limits and inequalities).