Follow-up from last time

Are these sequences increasing? decreasing? monotonic? bounded? convergent? Cauchy?

(a) $x_n = 1/n$
Bounded, monotonically decreasing, Cauchy, converges to 0.

(b) $x_n = \cos(\pi n)$
Bounded, not monotonic, divergent, not Cauchy.

(c) $x_n = 2^n$
Monotonically increasing, unbounded, divergent, not Cauchy.

(d) $x_n = 2^{1/n}$
Monotonically decreasing, bounded, Cauchy, converges to 1.

(e) $x_n = \cos(n)$
Bounded, not monotonic, divergent, not Cauchy, not obvious.

Examples of subsequences

Suppose $x_n = \cos\left(\frac{n\pi}{2}\right)$. Do the following subsequences of $\{x_n\}_{n=1}^{\infty}$ converge or diverge?

- $\{x_{2k}\}_{k=1}^{\infty}$
  Alternating ± 1, so diverges.
- $\{x_{j^2}\}_{j=1}^{\infty}$ or 0, 1, 0, 1, ...
  Bounded but oscillating subsequence, so diverges.
- $\{x_{2^i}\}_{i=1}^{\infty}$ or -1, 1, 1, ...
  Bounded, (weakly) increasing, monotonic, convergent to limit 1, Cauchy sequence too.

Assignment due next class

- Write solutions to Exercises 2.1.15 and 2.1.20.
- Read subsection 2.2.2 in the textbook.