Reminder

The first exam takes place in class this Friday (February 16).

Material covered on the exam: Sections 0.3, 1.1–1.4, and 2.1–2.2.

Please bring your own paper to write on.

Summary of methods for proving convergence of sequences

1. the definition of limit
2. monotone convergence theorem
3. squeeze theorem
4. geometric sequences
5. comparison test
6. ratio test

Lemma 2.2.1

Theorem (Squeeze theorem, or sandwich theorem)

If $\forall n\ (a_n \le x_n \le b_n)$*, and if* $\lim_{n\to\infty} a_n = L = \lim_{n\to\infty} b_n$*, then* $\lim_{n\to\infty} x_n$ *exists and equals* L*.*

Variation of the proof in the book.

Fix a positive number ε. Since $\lim_{n\to\infty} a_n = L$, there exists M_a such that $|a_n - L| < \varepsilon$ when $n \ge M_a$. Since $\lim_{n\to\infty} b_n = L$, there exists M_b such that $|b_n - L| < \varepsilon$ when $n \ge M_b$.
Let M denote $\max(M_a, M_b)$. If $n \ge M$, then

$$L - \varepsilon < a_n \le x_n \le b_n < L + \varepsilon.$$

Thus $|x_n - L| < \varepsilon$ when $n \ge M$. □

Assignment due next class

Make a list of the main concepts and theorems from sections 0.3, 1.1–1.4, and 2.1–2.2. [not to hand in]