Three views of the derivative

Suppose $f\colon I \to \mathbb{R}$ (where I is an interval), and c is an interior point of I. Then f is *differentiable at c* when any of the following equivalent properties holds.

1. The limit $\lim_{x\to c} \dfrac{f(x) - f(c)}{x - c}$ exists, in which case the limit is the *derivative*, usually denoted by $f'(c)$.
2. There exists a function $F_c\colon I \to \mathbb{R}$ with the properties that F_c is continuous at c and $f(x) = F_c(x)(x - c) + f(c)$ for every x.
   In this situation, $f'(c) = F_c(c)$.
3. There exists a "best linear approximation" of f at c: namely, a linear function T with the property that

$$\lim_{h\to 0} \frac{f(c+h) - f(c) - T(h)}{h} = 0.$$

   Here $T(h)$ has to be the linear function $h \to f'(c)h$.

Assignment due next class

Write a solution to Exercise 4.1.5.