1. Compute $\dfrac{\partial^2}{\partial x \partial y} f(x - y, xy)$.

2. Define each of the following terms:

   (a) $f(x, y) \in C^1$;

   (b) $f(x, y)$ is homogeneous of degree n;

   (c) the directional derivative of $f(x, y)$ in the direction ξ_α at a point (a, b).

3. State

   (a) the mean-value theorem for functions of two variables;

   (b) any other interesting theorem from this course.

4. At the point $(2, 1)$, the function $f(x, y) = xy + x \log y$ changes most rapidly in which direction? (You may specify the direction either as a vector (v_1, v_2) or as an angle α.)

5. Give an example of a function $f(x, y)$ defined on $\mathbb{R}^2$ such that the partial derivatives $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ exist at all points, but f is not continuous at the origin. Explain why your example works.

6. (a) Assume that the pair of equations

$$u^2 + v^2 - xy = 1$$
$$ux - vy + uv = \frac{1}{2}$$

   determines dependent variables u and v implicitly as functions of independent variables x and y. Find $\dfrac{\partial u}{\partial x}$.

   (b) Discuss the validity of the assumption in part (a) near the point where $x = 0$, $y = 0$, $u = 1/\sqrt{2}$, $v = 1/\sqrt{2}$.

7. (a) Find the Taylor expansion of $xy + \cos(2y)$ in powers of $(x - 2)$ and $(y - \pi)$ through the quadratic terms.

   (b) Write the remainder in part (a) in as simple a form as you can.