

Each part of each problem counts 5 points.

1. Let $X = \{a, b, c\}$ and $\mathcal{T} = \{\emptyset, X, \{a\}, \{b, c\}\}$.
 - (i) List the \mathcal{T} -closed subsets of the space X .
 - (ii) Find $\text{Int}(\{b\})$, the interior of the set $\{b\}$.
 - (iii) Find $\text{Cl}(\{b\})$, the closure of the set $\{b\}$.
 - (iv) Find $\text{Bd}(\{b\})$, the boundary of the set $\{b\}$.

Now consider the subspace $A = \{a, b\}$ with the relative topology \mathcal{T}_A . Answer the previous four questions for the topological space (A, \mathcal{T}_A) : namely,

 - (v) List the \mathcal{T}_A -closed subsets of the space A .
 - (vi) Find $\text{Int}_A(\{b\})$, the interior of the set $\{b\}$ relative to A .
 - (vii) Find $\text{Cl}_A(\{b\})$, the closure of the set $\{b\}$ relative to A .
 - (viii) Find $\text{Bd}_A(\{b\})$, the boundary of the set $\{b\}$ relative to A .
2. Consider the function $f : \mathbb{R} \rightarrow \mathbb{R}$ given by the formula $f(x) = |x|$ (the absolute value function). Is this function

(i) \mathcal{C} - \mathcal{C} continuous?	(ii) \mathcal{H} - \mathcal{H} continuous?
(iii) \mathcal{H} - \mathcal{C} continuous?	(iv) \mathcal{U} - \mathcal{D} continuous?

In each case justify your answer. Recall that \mathcal{C} denotes the open half-line topology, \mathcal{H} denotes the half-open interval topology, \mathcal{U} denotes the usual topology, and \mathcal{D} denotes the discrete topology.
3. Define the following terms:

(i) interior of a set	(ii) limit point of a set
(iii) homeomorphism	(iv) open function

In the next four problems, give a brief explanation if the answer is “Yes”, and find a counterexample if the answer is “No”.

4. Is every set in a topological space the union of an open set and a closed set?
5. Is the set of closed intervals of the form $[a, b]$, where $a < b$, a base for some topology on the real numbers \mathbb{R} ?
6. If $f : X \rightarrow Y$ is a function, and V and W are subsets of X , must it be true that $f(V \cup W) = f(V) \cup f(W)$?
7. Is the boundary of a subset of a topological space always a closed set?

Extra credit (5 points):

8. Determine the set of all functions $f : \mathbb{R} \rightarrow \mathbb{R}$ that are \mathcal{C} - \mathcal{U} continuous.