

Question

If \mathcal{B} is some collection of subsets of X , and \mathcal{B} contains at least the sets \emptyset and X , is \mathcal{B} necessarily a basis for some topology on X ?

No.

Example

If $X = \mathbb{R}$, and \mathcal{B} consists of all unbounded intervals (a, ∞) and $(-\infty, b)$ (for all real numbers a and b), then \mathcal{B} is not a basis for any topology. Why not?

The intersection $(-\infty, 1) \cap (0, \infty)$ is the bounded interval $(0, 1)$, which is not a union of elements of \mathcal{B} .

Assignment due next class

1. Write solutions to number 5 in Exercises 1.2 and number 3 in Exercises 2.3.
2. Read section 3.1 in the textbook.