Convergence of sequences in metric spaces

In a metric space (X, d), a sequence x_1, x_2, ... *converges* to x if for every open ball B centered at x, the sequence is "eventually in B."

More precisely, for every (positive) radius r, there is some natural number $n_0(r)$ with the property that $x_n \in B_r(x)$ whenever $n \geq n_0$.

The emphasis is on r being small, so usually the Greek letter ε is used: namely, for every positive ε, there exists n_0 such that $d(x_n, x) < \varepsilon$ whenever $n \geq n_0$.

In metric spaces, sequences determine the topology.

1. A subset S of a metric space is closed if and only if the limit of every convergent sequence of points of S belongs to S.

2. A function f between metric spaces is continuous if and only if f maps convergent sequences to convergent sequences: namely, whenever $x_n \to x$ in the domain, $f(x_n) \to f(x)$ in the codomain.

Assignment due next class

Write a solution to one of problems 1, 2, 3, 4, 6, 8 in Exercises 6.2.