Exam results

- Scoring algorithm: (10 points per problem) $+$ 50.
- Class statistics: mean 81, median 80, maximum 106; there were four scores ≥ 100.

Examples: finite versus infinite

- The interval $(0,1)$ is the union $\bigcup_{0<x<1} (\frac{x}{2}, \frac{1+x}{2})$ of infinitely many open intervals, but is not the union of any finite subcollection of these intervals.
- The interval $[0,1]$ is the union $\bigcup_{0\le x\le 1} [\frac{x}{2}, \frac{1+x}{2}]$ of infinitely many closed intervals, and is also the union of a finite subcollection of these intervals: namely, $[\frac{0}{2}, \frac{1+0}{2}] \cup [\frac{1}{2}, \frac{1+1}{2}]$.
- $\bigcap_{n=1}^{k} (0, \frac{1}{n}) \neq \varnothing$ for each natural number k, yet $\bigcap_{n=1}^{\infty} (0, \frac{1}{n}) = \varnothing$.
- $\bigcap_{n=1}^{k} [0, \frac{1}{n}] \neq \varnothing$ for each natural number k, and $\bigcap_{n=1}^{\infty} [0, \frac{1}{n}] \neq \varnothing$.

Compactness

Definitions

An *open cover* of a subset S of a topological space is a collection of open sets whose union contains S.

A subset S of a topological space is *noncompact* when there is some open cover of S (consisting of infinitely many open sets) with the property that no finite number of those sets is an open cover of S.

And S is *compact* if every open cover can be reduced to a finite subcover.

Examples in $\mathbb{R}$ with the Euclidean topology

- $(0, 1)$ is noncompact by the first example above.
- $[0, \infty)$ is noncompact, because (for instance) the sets $(-1, n)$ cover $[0, \infty)$, but no finite subcollection of these open sets forms a cover.
- $[0, 1]$ is compact. (Not obvious; needs proof.)

Exercise

For which of the following topological spaces is the set of even natural numbers a compact subset?

1. $(\mathbb{N}, \text{discrete})$
2. $(\mathbb{N}, \text{indiscrete})$
3. $(\mathbb{N}, \text{initial segment})$
4. $(\mathbb{N}, \text{final segment})$
5. $(\mathbb{N}, \text{finite-closed})$