1. Evaluate the complex line integral

$$\int_\gamma \frac{(z-3)}{(z-1)(z-2)}\,dz,$$

where γ is the circular path defined by $\gamma(t) = \frac{3}{2}\,e^{2\pi i t}$, $0 \le t \le 1$.

2. Determine the radius of convergence of the gap series

$$\sum_{n=1}^{\infty} \frac{z^{n^2}}{3^n} = \frac{z}{3} + \frac{z^4}{9} + \frac{z^9}{27} + \cdots .$$

3. Suppose that f is a holomorphic function, and let u and v denote the real and imaginary parts of f. Prove that the product uv is a harmonic function.

4. Suppose f is a function that is holomorphic in the unit disk $D(0,1)$ and that satisfies the inequality $|f(z)| < 2$ when $|z| < 1$. If the power series expansion for f is given by

$$f(z) = \sum_{n=0}^{\infty} a_n z^n,$$

how big can the coefficient $|a_{617}|$ be?

5. A calculus student named Lee knows that since $(-2)^3 = -8$, the cube root of -8 is equal to -2. Lee is therefore puzzled to observe that the computer algebra system Maple responds to the input

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(-8.0)^(1/3);
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with the output

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1.000000000+1.732050807*I
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(which is a numerical approximation of $1+i\sqrt{3}$). Write an explanation to resolve Lee's perplexity. You may assume that Lee knows what complex numbers are.