Properties of harmonic functions

The goal of this exercise is to demonstrate that you understand and can apply the relationship between holomorphic and harmonic functions.

In a disk of radius r centered at the origin, a holomorphic function f can be reconstructed from its real part u via

$$f(z) = \frac{1}{2\pi} \int_0^{2\pi} \frac{re^{i\theta} + z}{re^{i\theta} - z} \, u(re^{i\theta}) \, d\theta + iv(0), \qquad |z| < r,$$

where v is the imaginary part of f. Equivalently, a harmonic function u in a disk of radius r centered at the origin can be represented by the Poisson integral:

$$u(z) = \frac{1}{2\pi} \int_0^{2\pi} \frac{r^2 - |z|^2}{|re^{i\theta} - z|^2} \, u(re^{i\theta}) \, d\theta, \qquad |z| < r.$$

Many important local properties of holomorphic functions follow from the Cauchy integral formula. In view of the two integral formulas above, it is reasonable to expect that harmonic functions should have many local properties analogous to those of holomorphic functions.

Which of the following properties of holomorphic functions actually do carry over to harmonic functions?

1. A holomorphic function is infinitely differentiable.
2. A holomorphic function has a (local) power series expansion.
3. The derivatives of a holomorphic function at the center of a disk can be estimated in terms of the maximum of the modulus of the function on the boundary of the disk (Cauchy's estimates).
4. Holomorphic functions have the mean value property.
5. Holomorphic functions satisfy a maximum principle.
6. Two holomorphic functions that agree on a sufficiently big set agree identically.
7. The range of a nonconstant holomorphic function is open.
8. A family of holomorphic functions is normal if and only if it is locally bounded.
9. A property of your choosing.