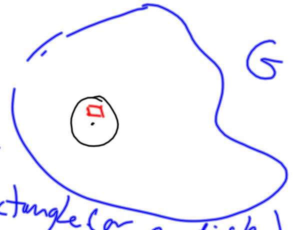
Theorem (Morera)**.** *Suppose a function f on an open set G has the following properties:*

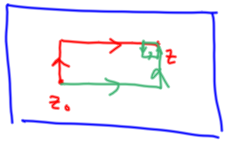
1. *The function f is continuous.*
2. *Every point in G has some neighborhood such that for every rectangle R lying in the neighborhood, $\int_{\partial R} f(z)\,dz = 0$.*

Then f is analytic in G.



Proof. Conclusion is local, so WLOG assume G is a small rectangle (or a disk).

Goal: show that f has an analytic anti-derivative.



Define $F(z) = \int f(w)\,dw$

over a path that goes first vertically and then horizontally, ending at z.

By fundamental theorem of calculus, $\frac{\partial F}{\partial x}$ exists and equals $f(z)$.

To find $\frac{\partial F}{\partial y}$, make a rectangular detour.

Again apply fundamental theorem of calculus (and $dz = i\,dy$) to get

$\frac{\partial F}{\partial y} = i f(z)$. F satisfies Cauchy-Riemann equations.

F is analytic.

$F' = f$, so f is analytic.

Reminder: Exam 2 takes place Thursday, November 5