# Examination 2

1. State the following theorems: Liouville's theorem about entire functions, Morera's theorem, and the Casorati–Weierstrass theorem.

2. Give an example of an open set, an analytic function $f$ defined on the set, and two paths $\gamma_1$ and $\gamma_2$ in the set having the same endpoints [in other words, $\gamma_1(0) = \gamma_2(0)$ and $\gamma_1(1) = \gamma_2(1)$] such that $\int_{\gamma_1} f(z)\,dz \neq \int_{\gamma_2} f(z)\,dz$.

3. Determine (with proof) the maximum value and the minimum value of the real-valued expression $\left|z^2 - 1\right|$ when $|z| \leq 1$.

4. Suppose $f$ is analytic in $\{\, z \in \mathbb{C} : 0 < |z| < 1 \,\}$, the punctured unit disk. If $f$ has a removable singularity at the origin, then what can you say about the singularity of $1/f$, the reciprocal function?

5. Does there exist an entire function $f$ such that $f(n) = n \cdot (-1)^n$ for every natural number $n$?

6. Determine the residue of the rational function $\dfrac{1}{(z^2-1)^5}$ at the point where $z = 1$.