Math 650-600: Several Complex Variables

Harold P. Boas
boas@tamu.edu

Announcement

Math Club Meeting

Monday, January 31 at 6:30 pm

Blocker 627

Robert S. Smith (Miami University)

Boolean Algebra and Switching Circuits

Announcement

Next week I will be traveling to Washington.

Our class *will meet* on February 1 (Tuesday)
but *will not meet* on February 3 (Thursday).

Reminders from last time

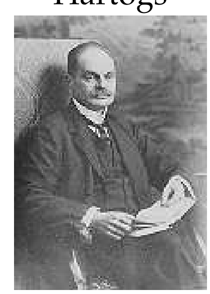
Convergence domains for power series are logarithmically convex complete Reinhardt (multicircular) domains.

A function with continuous (real) partial derivatives is holomorphic if (a) it is represented locally by an absolutely convergent power series, or (b) is represented locally by the iterated Cauchy integral formula, or (c) satisfies the Cauchy-Riemann equations in each variable separately.

A technical result

Theorem (Hartogs). If $f : \Omega \to \mathbb{C}$ is holomorphic in each variable separately (with the other variables held fixed), then f is holomorphic.
In particular, a separately holomorphic function is automatically continuous in all variables jointly.

Friedrich Hartogs

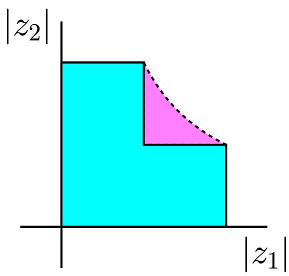


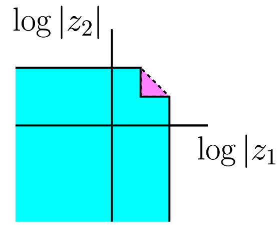
(1874-1943)

Proofs may be found in the books by Hörmander, by Krantz, and by Narasimhan.

The Hartogs phenomenon: version 1

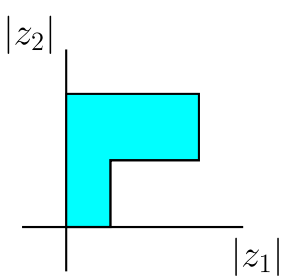
Every holomorphic function in a complete Reinhardt domain automatically extends to be a holomorphic function on the logarithmically convex envelope of the given domain.



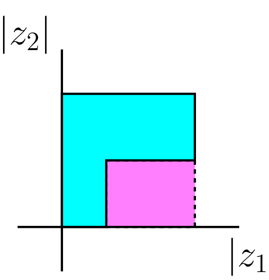


The Hartogs phenomenon: version 2

Every holomorphic function in a connected (not necessarily complete) Reinhardt domain containing 0 automatically extends to be a holomorphic function on the logarithmically convex complete envelope of the given domain.



A "Hartogs figure"



The envelope

Proof of version 2

Notation: for each point w in the given domain, let T_w denote the torus generated by w: namely, the set of points $(e^{i\theta_1}w_1, \ldots, e^{i\theta_n}w_n)$ as the angles $\theta_1, \ldots, \theta_n$ vary independently.

Given a holomorphic function f, form its iterated Cauchy integral over the torus T_w.

This integral is independent of w because the set of points w for which the integral agrees with f near the origin is non-empty, open, and closed.

Thus the iterated Cauchy integral defines a holomorphic extension of the function to the "complete envelope".
From version 1, we already know that the function extends from the complete envelope to the logarithmically convex envelope.