Math 650-600: Several Complex Variables

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Exercise from last time

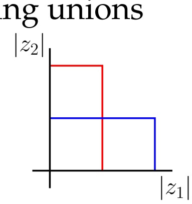
A domain Ω in $\mathbb{C}^n$ is holomorphically convex if and only if for every sequence of points in Ω with no accumulation point in Ω there is a holomorphic function on Ω that is unbounded on the sequence.

Set operations

Which of the following classes of domains
(a) convex domains, (b) domains of holomorphy, (c) polynomially convex domains, (d) monomially convex domains, (e) weakly linearly convex domains
are preserved under the following operations?

(i) taking Cartesian products
All, because $\mathrm{hull}_{\Omega_1\times\Omega_2}(K_1 \times K_2) \subseteq \mathrm{hull}_{\Omega_1}(K_1) \times \mathrm{hull}_{\Omega_2}(K_2)$.
(ii) taking unions

None:



(iii) taking the union of an increasing sequence
All, but in some cases this is non-trivial.

Behnke-Stein theorem

To be proved later: *The union of an increasing sequence of domains of holomorphy is a domain of holomorphy.*
[H. Behnke and K. Stein, Konvergente Folgen von Regularitätsbereichen und die Meromorphiekonvexität, *Mathematische Annalen* **116** (1938) 204–216.]

Heinrich Behnke

1898–1979

Karl Stein

1913–2000

Set operations, continued

Which of the following classes of domains
(a) convex domains, (b) domains of holomorphy, (c) polynomially convex domains, (d) monomially convex domains, (e) weakly linearly convex domains
are preserved under the following operations?

(iv) taking a connected component of the interior of an intersection

All.