$\in$ means is an element of SUBSET

WEEK 6 REVIEW: SETS and MULTIPLICATION PRINCIPLE

$1 \in A$, $\{1\} \subseteq A$

Example

such that

Let $U = \{x \mid x \text{ is a postive integer less than } 8\}$, $A = \{1, 2, 3, 4\}$, $B = \{3, 4, 5\}$, and $C = \{5, 6, 7\}$

a. Write U in roster notation $U = \{1, 2, 3, 4, 5, 6, 7\}$

b. $A^c = \{x \mid x \in U \text{ and } x \notin A\} = \{5, 6, 7\}$

c. $A \cap B = \underbrace{\{x \mid x \in A \text{ and } x \in B\}}_{\text{set builder}} = \underbrace{\{3, 4\}}_{\text{roster}}$

d. $A \cup B = \{x \mid x \in A \text{ or } x \in B\} = \{1, 2, 3, 4, 5\}$

e. List all the subsets of C $n(C) = 3$, $2^3 = 8$

proper subsets

$\{5, 6, 7\}$, $\boxed{\emptyset, \{5\}, \{6\}, \{7\}, \{5, 6\}, \{5, 7\}, \{6, 7\}}$

# of subsets of a set with n elements is 2^n

A proper set must have fewer elements

Determine if the statements below are true or false

Two sets are disjoint if the intersection is the empty set

TRUE f. A and C are disjoint sets

$A \cap C = \emptyset = \{\ \}$

False g. $\emptyset \in A$ $\emptyset \subseteq A$ is true $1 \in A$

false h. $\{5, 6, 7\} \subset C$

proper subset

$1 \leq 1$

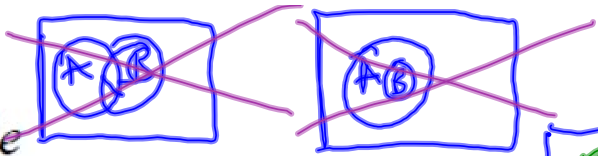
$0 < 1$

$\{5, 6, 7\} \subseteq C$ TRUE

$\{6, 7\} \subset C$ TRUE

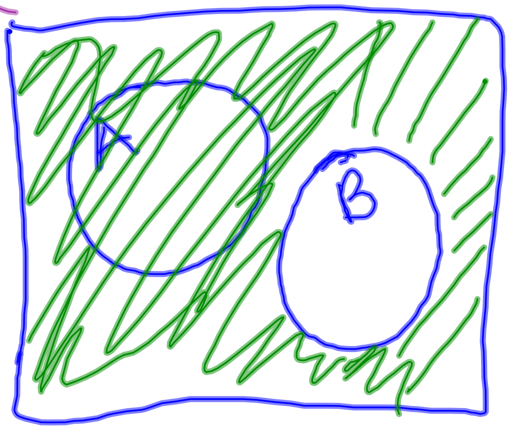
$\{7\} \subset C$ TRUE

$\emptyset \subset C$ TRUE

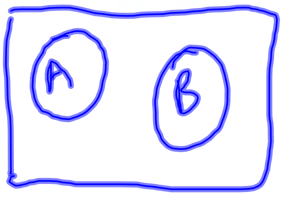


Example

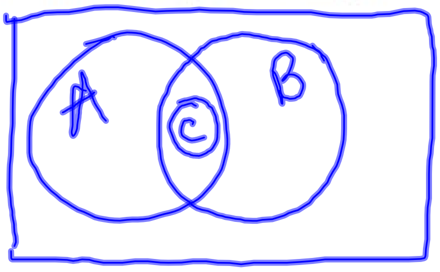
Use Venn diagrams to indicate



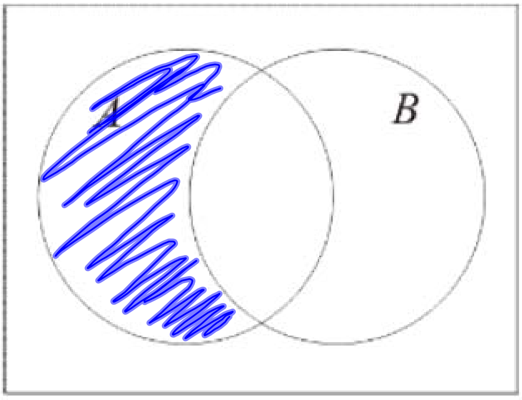
a. $A \subset U, B \subset U, A \subset B^c$



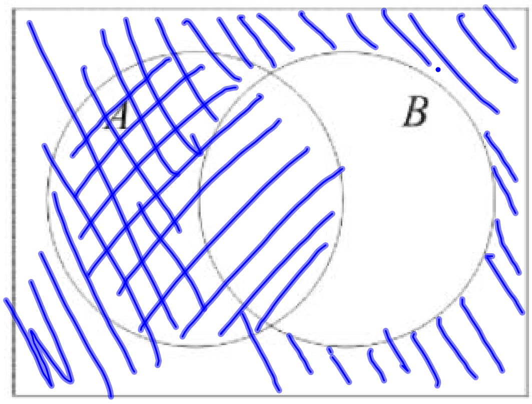
b. $A \subset U, B \subset U, C \subset U, C \subset A \cap B$



Example
Shade the indicated regions on the Venn diagram

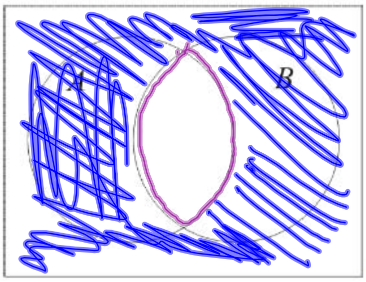


a. $A \cap B^c$



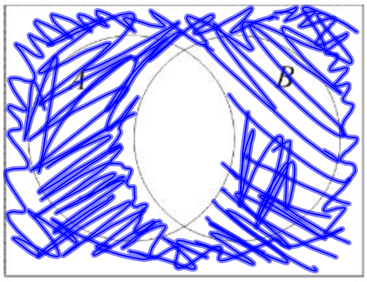
join

b. $A \cup B^c$



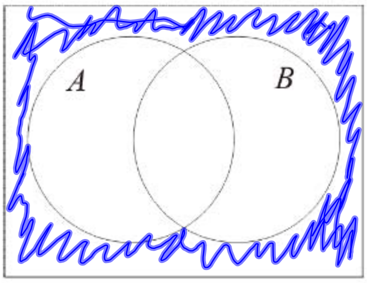
c. $(A \cap B)^c$

Same



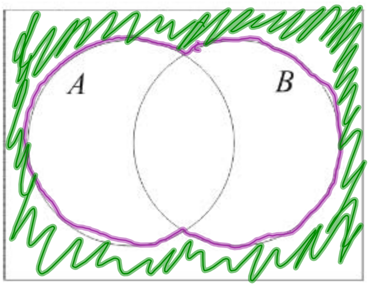
join

d. $A^c \cup B^c$



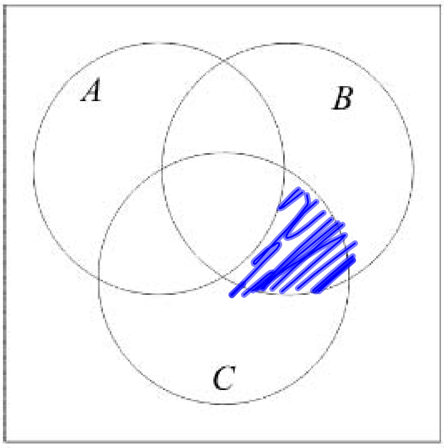
e. $A^c \cap B^c$

Same

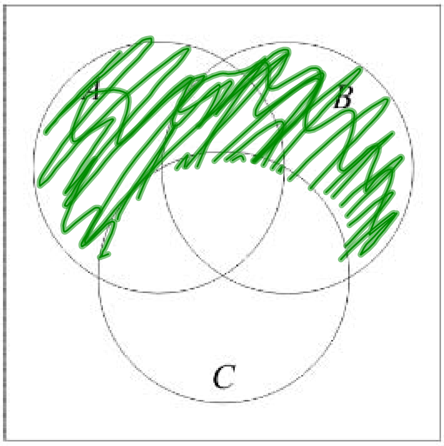


f. $(A \cup B)^c$

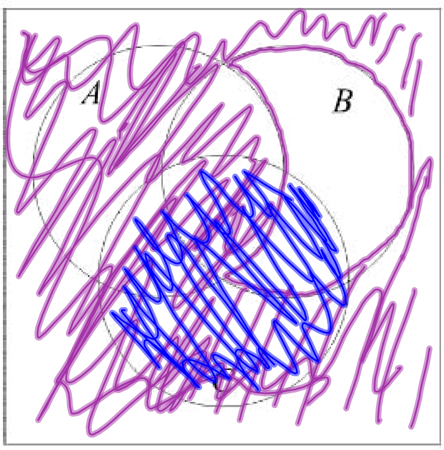
$A^c \cup B^c$ (d)



g. $A^c \cap B \cap C$

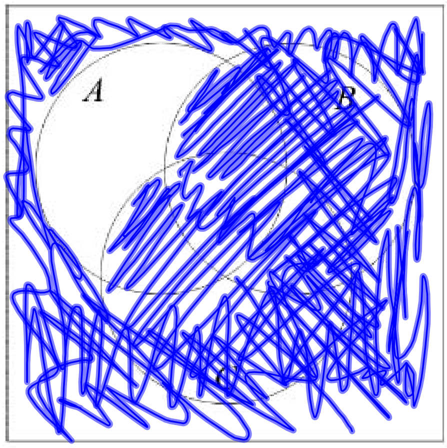


h. $(A \cup B) \cap C^c$

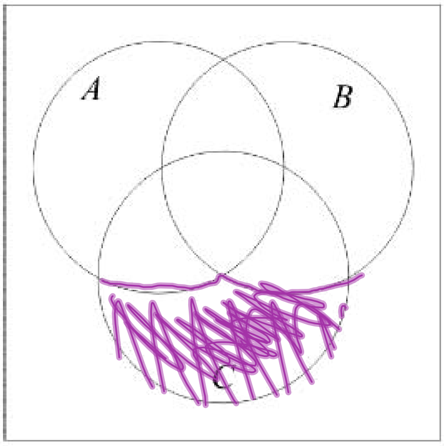


joining

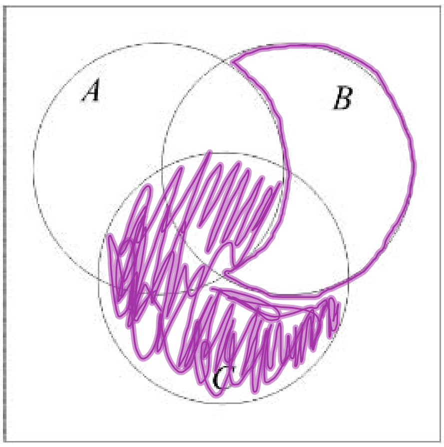
i. $(A^c \cap B)^c \cup C$



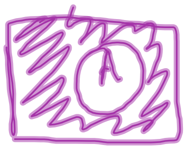
j. $A^c \cup B \cup C$



k. $(A \cup B)^c \cap C$



l. $(A^c \cap B)^c \cap C$



Example

Let U be the set of all staff at Texas A&M University and let

$A = \{x|x \text{ owns an automobile}\}$

$H = \{x|x \text{ owns a house}\}$

$P = \{x|x \text{ owns a piano}\}$

Describe the following sets in words

a. A^c

The staff at TAMU that does not own an automobile.

b. $A \cap H^c$

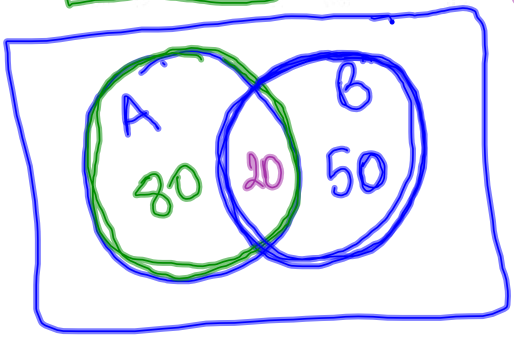
The staff at TAMU that own an automobile but not a house.

c. $A^c \cup P^c$ The staff at TAMU who do not have an automobile or do not have a piano.

d. $A^c \cap H^c \cap P^c$ The staff at TAMU that don't have an automobile and don't have a house and don't have a piano

Example

If $n(A)=100$, $n(A\cap B)=20$, and $n(A\cup B)=150$, what is $n(B)$?



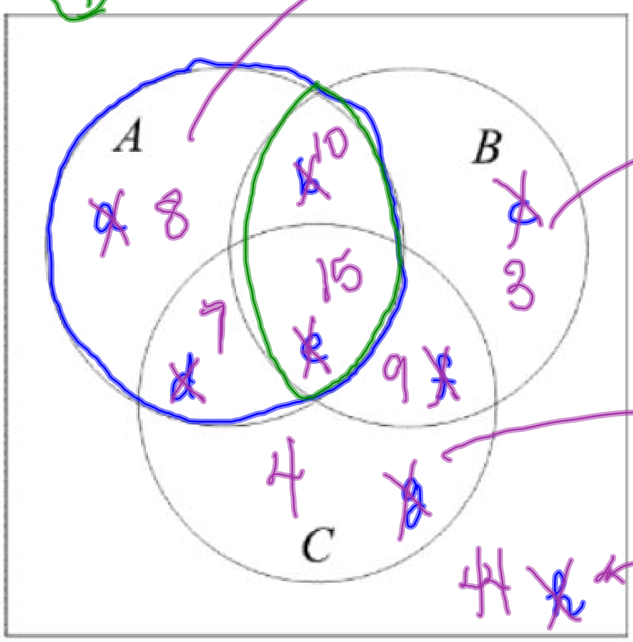
$n(B)=70$

Example

Given $n(U)=100$, $n(A)=40$, $n(B)=37$, $n(C)=35$, $\boxed{n(A\cap B)=\cancel{25}}$, $n(A\cap C)=22$, $n(B\cap C)=24$, and $n(A\cap B\cap C^c)=10$, find $n(A^c\cap B\cap C)$.

9

40 − 10 − 15 − 7



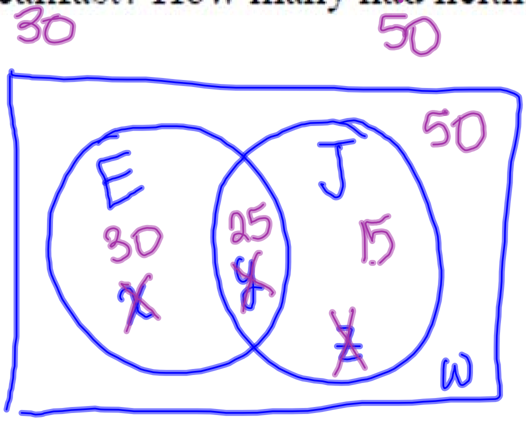
37 − 10 − 15 − 9

35 − 7 − 15 − 9

100 − 8 − 10 − 3 − 7 − 15 − 9 − 4

Example

In a survey of 120 adults, 55 said they had an egg for breakfast that morning, 40 said they had juice for breakfast, and 70 said they had an egg or juice for breakfast. How many had an egg but no juice for breakfast? How many had neither an egg nor juice for breakfast?



$n(J) = 40 = y + z$

$n(E) = 55 = x + y$

$n(\mathcal{U}) = 120 = x + y + z + w$

$\underline{n(E \cup J) = 70 = x + y + z}$

$70 = 55 + 40 - n(E \cap J)$

$n(E \cap J) = 95 - 70$
$= 25$

UNION RULE

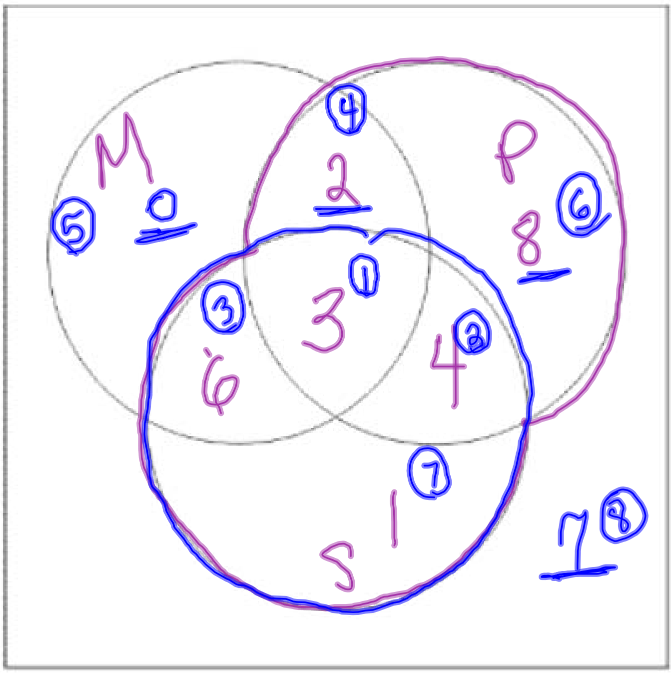
$$n(E \cup J) = x + y + z = \underbrace{(x+y)}_{n(E)} + \underbrace{(y+z)}_{n(J)} - \underbrace{y}_{n(E \cap J)}$$

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

Example

Determine how many pizzas were sold if

1. • 3 pizzas had mushrooms, pepperoni, and sausage
2. • 7 pizzas had pepperoni and sausage $7-3=4$
3. • 6 pizzas had mushrooms and sausage but not pepperoni
4. • 15 pizzas had two or more of these toppings
5. • 11 pizzas had mushrooms
6. • 8 pizzas had only pepperoni
7. • 24 pizzas had sausage or pepperoni
8. • 17 pizzas did not have sausage



Let

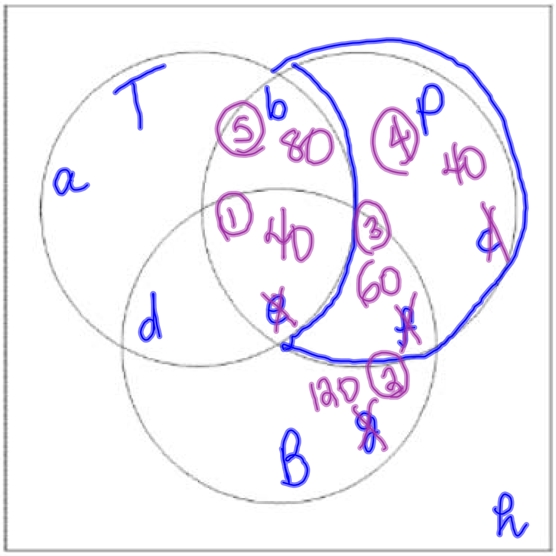
$M = \{x \mid x \text{ is a pizza with mushroom}\}$

$P = \{x \mid x \text{ is a pizza with pepperoni}\}$

$S = \{x \mid x \text{ is a pizza with sausage}\}$

Example

Six hundred people were surveyed and it was found that during the past year, 330 did not travel by bus, 100 traveled by plane but not by train, 150 traveled by train but not by plane, 120 traveled by bus but not by train or plane, 100 traveled by both bus and plane, 40 traveled by all three, and 220 traveled by plane. How many did not travel by any of these three modes of transportation?



$n(\mathcal{U}) = 600$

$n(B^c) = 330$

$n(P \cap T^c) = 100 = f + c$ (4)

$n(T \cap P^c) = 150$

$n(B \cap (T \cup P)^c) = 120$ ✓ (2)

$n(B \cap P) = 100$ (3) ✓

$n(B \cap T \cap P) = 40$ ✓ (1)

$n(P) = 220$ (5)

$$600 = a + \underset{80}{\cancel{b}} + \underset{40}{\cancel{c}} + d + \underset{40}{\cancel{e}} + \underset{60}{\cancel{f}} + \underset{120}{\cancel{g}} + h$$

$$330 = a + \underset{80}{\cancel{b}} + \underset{40}{\cancel{c}} + h$$

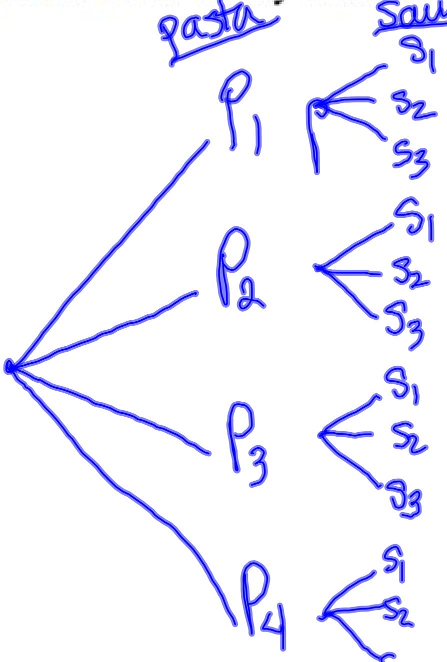
$$150 = a + d$$

$$\begin{aligned} a + d + h &= 260 \\ a \quad\; + h &= 210 \\ a + d \quad\; &= 150 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 1 & 1 & 1 & 260 \\ 1 & 0 & 1 & 210 \\ 1 & 1 & 0 & 150 \end{array}\right] \xrightarrow{\text{rref}}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 100 \\ 0 & 1 & 0 & 50 \\ 0 & 0 & 1 & 110 \end{array}\right] \quad \begin{aligned} a &= 100 \\ d &= 50 \\ h &= 110 \end{aligned}$$

Example

At a pasta diner there is a choice of 4 different pastas and 3 different sauces. How many dinners can be made?



outcom

$P_1 S_1$

$P_1 S_2$

$P_1 S_3$

$\vdots$

choose pasta AND sauce

$P_4 S_3$

$$\underset{\text{pasta}}{\underline{4}} \cdot \underset{\text{sauce}}{\underline{3}} = 12$$

$$S = \{P_1S_1, P_1S_2, P_1S_3, \ldots\ P_4S_3\}$$

Example

How many different 4-digit access codes can be made if

a. there are no restrictions?
b. there are no repeats?
c. the first digit cannot be a 0 or a 1 and no repeats are allowed?
d. four of the same digit is not allowed?

0, 1, 2, 3, ~~4~~, 5, 6, 7, 8, 9

a) $\underset{1^{st}\text{ Digit}}{\underline{10}} \cdot \underset{2^{nd}}{\underline{10}} \cdot \underset{3^{rd}}{\underline{10}} \cdot \underset{4^{th}}{\underline{10}} = 10{,}000$

b) $\underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} = 5040$

c) $\underline{8} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} = 4032$

d) $\underline{10} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10} - \underline{10} = 9990$

4444 NOT OK 4443, 4434, 4344, 3444 are ok

A A A A C C C

Example
A minivan can hold 7 passengers. An adult must sit in one of the two front seats and anyone can sit in the rear 5 seats. A group of 4 adults and 3 children are to be seated in the van. How many different seating arrangements are possible?

$\underset{F}{\underline{4}} \cdot \underset{F}{\underline{3}} \quad \underset{B}{\underline{5}} \quad \underset{B}{\underline{4}} \quad \underset{B}{\underline{3}} \quad \underset{B}{\underline{2}} \quad \underset{B}{\underline{1}}$

$= 12!$ MATH → PRB → 4: !

No restrictions $\underline{12} \cdot \underline{11} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1}$

$= 479{,}001{,}600$

Example
You have a class of 12 children, 6 boys and 6 girls. How many ways can the children be seated in a row

a. if boys and girls must alternate?
b. if a girl must be seated at each end?

$\underline{12} \cdot \underline{6} \cdot \underline{5} \cdot \underline{5} \cdot \underline{4} \cdot \underline{4} \cdot \underline{3} \cdot \underline{3} \cdot \underline{2} \cdot \underline{2} \cdot \underline{1} \cdot \underline{1}$

B or G

~~B~~ ~~B~~ B B B B ~~G~~ ~~G~~ G G G G

# of ways with a boy seated first + # of ways with a girl seated first

$\underset{B}{\underline{6}} \cdot \underset{G}{\underline{6}} \cdot \underset{B}{\underline{5}} \cdot \underset{G}{\underline{5}} \cdot \_ \_ \_ \_ \_ \_ \_ \_$

+

$\underset{G}{\underline{6}} \cdot \underset{B}{\underline{6}} \cdot \underset{G}{\underline{5}} \cdot \underset{B}{\underline{5}} \cdot \_ \_ \_ \_ \_ \_ \_ \_$

$= 1{,}036{,}800$

b

$\underset{G}{\underline{6}} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7} \cdot \underline{6} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \cdot \underset{G}{\underline{5}}$

Example
You take a multiple choice test with 3 questions and each question has 5 possible answers. How many ways can the test be answered?

$\underset{Q1}{\underline{5}} \cdot \underset{Q2}{\underline{5}} \cdot \underset{Q3}{\underline{5}} = 125$

M J

Example

Matthew and Jennifer go to the movies with four of their friends. How many ways can these six children be seated if

a. there are no restrictions?

b. Matthew and Jennifer are seated next to each other?

c. Matthew and Jennifer are not next to each other?

a) $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

b) $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot \underset{\substack{M\&J \\ J\&M}}{2} = 240$

MJ

c) $720 - 240 = 480$

Example

Four couples are going to the movie together. How many ways can these eight people be seated if couples sit together?

C1 C2 C3 C4

$\underbrace{3 \cdot 2 \cdot 1}_{3!}$

$4 \cdot 3 \cdot 2 \cdot 1 \cdot \underset{C1}{2} \cdot \underset{C2}{2} \cdot \underset{C3}{2} \cdot \underset{C4}{\cancel{2}}$