WEEK 9 REVIEW – Probability

$$P(E) = \frac{n(E)}{n(S)}$$

Example

You are dealt 4 cards from a standard deck of 52 cards. Find the probability distribution table for the number of spades in your hand of 4 cards.

$n(S) = C(52,4) = 270,725$

EVENT	$n(E)$	$P(E) = n(E)/n(S)$
0 Spades	$\underbrace{C(13,0)}_{\text{0 spades}} \cdot \underbrace{C(39,4)}_{\text{4 non-spades}} = 82,251$	$\frac{82,251}{270,725} \approx 0.3038$
1 Spade	$C(13,1) \cdot C(39,3) = 118,807$	$\frac{118,807}{270,725} \approx 0.4388$
2 Spades	$C(13,2) \cdot C(39,2) = 57,798$	$\frac{57,798}{270,725} \approx 0.2135$
3 Spades	$C(13,3) \cdot C(39,1) = 11,154$	$\frac{11,154}{270,725} \approx 0.0412$
4 Spades	$C(13,4)\,C(39,0) = 715$	$\frac{715}{270,725} \approx 0.0026$

Example

A box has 30 transistors and a sample of 5 is chosen for testing to decide if the box is "good" or "bad". A box is considered "bad" if one or more transistors in the sample are found to be defective. What is the probability that a box that has 4 defective transistors will be considered "good"?

$$\frac{C(\overset{\text{Bad}}{4},0)\,C(\overset{\text{Good}}{26},5)}{C(30,5)} = \frac{65{,}780}{142{,}506} \approx 0.4616$$

Example

A bowl has 15 red jelly beans and 8 green jelly beans. A sample of 6 is chosen at random from the bowl. What is the probability that the sample will have at least one green jelly bean?

$$1 = P(0G) + \underbrace{P(1G) + P(2G) + P(3G) + P(4G) + P(5G) + P(6G)}$$

$$P(\text{at least } 1G) = 1 - P(\underset{\text{green}}{0G})$$

$$= 1 - \frac{C(8,0)\,\overset{\text{red}}{C(15,6)}}{C(23,6)} = 1 - \frac{5005}{100{,}947}$$

$$= \frac{1246}{1311} \approx 0.9504$$

Example

Matthew is studying for a Latin quiz and he learns the meaning of 24 nouns from the list of 30. The Latin quiz has 12 nouns. If a passing grade is 9 or more, what is the probability that Matthew passes this Latin quiz?

Know 9 or 10 or 11 or 12 nouns

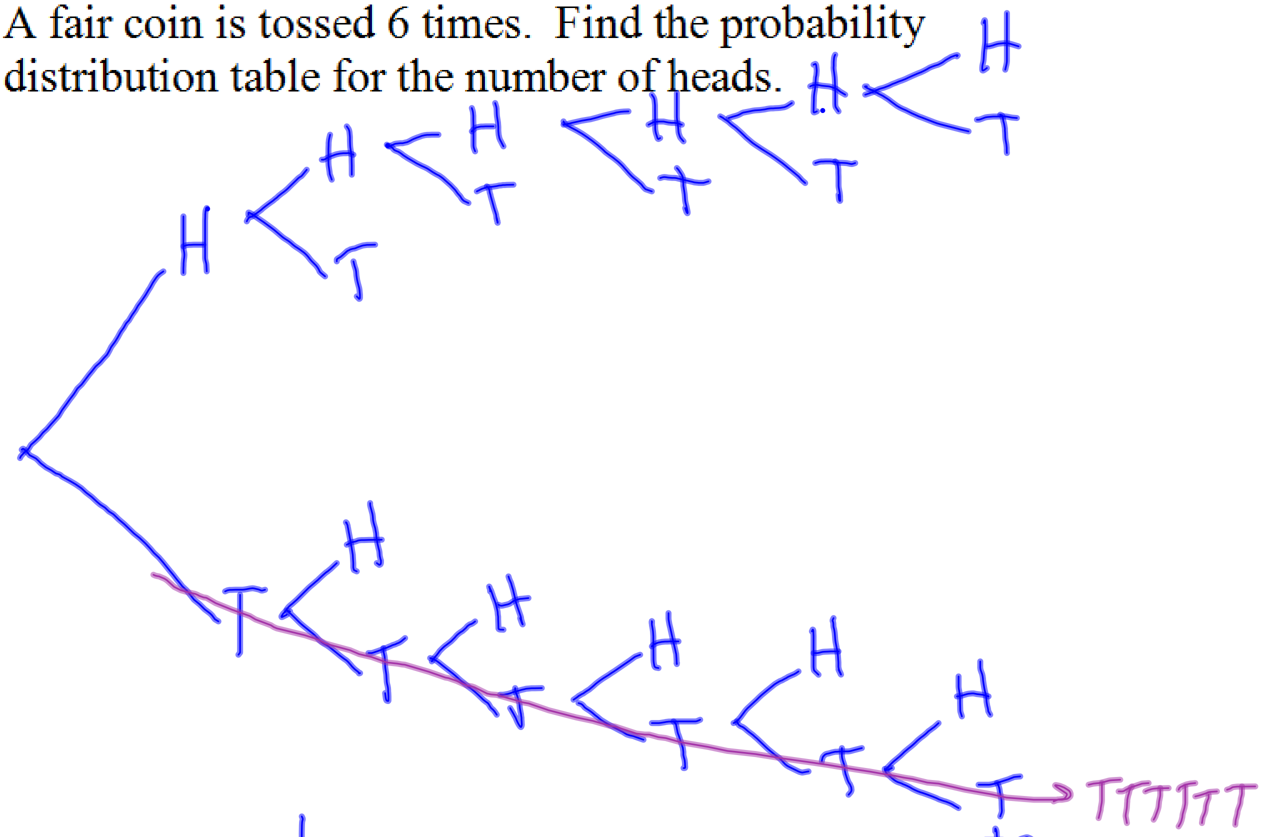
$$\frac{\overset{\text{Know}}{C(24,9)}\,\overset{\text{Don't Know}}{C(6,3)} + C(24,10)C(6,2) + C(24,11)C(6,1) + C(24,12)C(6,0)}{C(30,12)}$$

$$= \frac{73,249,940}{86,493,225} \approx 0.8469$$

$$\frac{2}{1^{st}} \cdot \frac{2}{2^{nd}} \cdot \frac{2}{3^{rd}} \cdot \frac{2}{4^{th}} \cdot \frac{2}{5^{th}} \cdot \frac{2}{6^{th}} = 64$$

Example

A fair coin is tossed 6 times. Find the probability distribution table for the number of heads.



EVENT	PROB
0 H	$C(6,0)/64 = 1/64$
1 H	$C(6,1)/64 = 6/64$
2 H	$C(6,2)/64 = 15/64$
3 H	$C(6,3)/64 = 20/64$
4 H	$C(6,4)/64 = 15/64$
5 H	$C(6,5)/64 = 6/64$
6 H	$C(6,6)/64 = 1/64$

6 spots

HTTTTT
THTTTT
TTHTTT
⋮

$$P(E) = \frac{n(E)}{n(S)}$$

Example

A bowl has 15 pieces of fruit and 5 of the pieces of fruit are rotten. There are 9 apples (3 are rotten) and 6 oranges. (2 are rotten)

(a) What is the probability that a randomly selected piece of fruit is a rotten orange?

(b) What is the probability that an apple is rotten?

(c) What is the probability that a good piece of fruit is an orange?

	Rotten	Good	TOTAL
APPLE	3	6	9
ORANGE	2	4	6
TOTAL	5	10	15

a) $P(R \cap O) = \frac{2}{15} = \frac{n(R \cap O)}{n(S)}$

b) "that an apple" → we know it is an apple

$$\underset{\uparrow \text{given that}}{P(R \mid A)} = \frac{n(A \cap R)}{n(A)} = \frac{3}{9} \left(= \frac{1}{3}\right)$$

c) $P(O \mid G) = \frac{n(O \cap G)}{n(G)} = \frac{4}{10}$

$$= \frac{n(O \cap G)/n(S)}{n(G)/n(S)} = \frac{P(O \cap G)}{P(G)}$$

$$\boxed{P(E \mid F) = \frac{P(E \cap F)}{P(F)}}$$

Example

Two fair five-sided dice are rolled.

(a) What is the probability that the sum is greater than 6?

(b) What is the probability that the sum is greater than 6 if at least one 3 is shown? ← know at least 1 three is shown

(c) What is the probability that the sum is 4 if both dice display the same number?

1-1	2-1	3-1	4-1	5-1
1-2	2-2	3-2	4-2	5-2
1-3	2-3	3-3	4-3	5-3
1-4	2-4	3-4	4-4	5-4
1-5	2-5	3-5	4-5	5-5

(a) $P(\text{sum} > 6) = 10/25$

(b) new sample space has 9 outcomes

$\frac{4}{9}$

(c) $S = \{1\text{-}1, 2\text{-}2, 3\text{-}3, 4\text{-}4, 5\text{-}5\}$

$P = \frac{1}{5}$

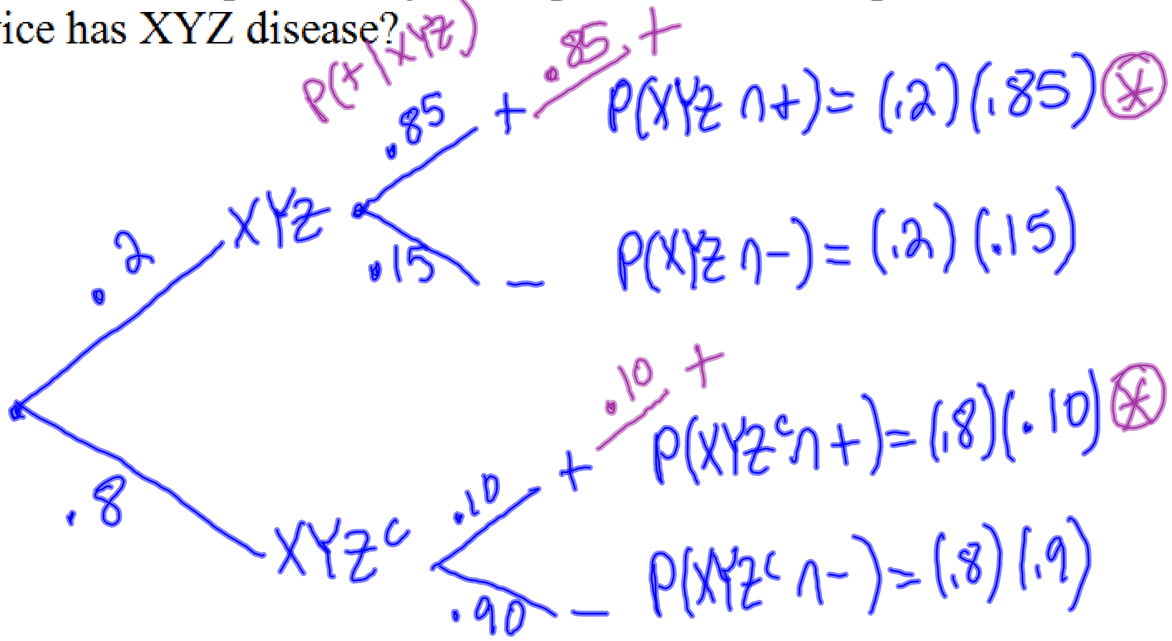
Example

A local clinic tests for XYZ disease. It is known that 20% of the patients coming to the clinic have XYZ disease. The test for XYZ is positive for 85% of the patients that have XYZ and it is positive for 10% of the patients that do not have this disease.

a) Represent this experiment in a tree diagram.

b) What is the probability that a person who tests positive has XYZ disease? $P(XYZ \mid +)$

c) What is the probability that a person who tests positive twice has XYZ disease?



$$P(XYZ \mid +) = \frac{P(XYZ \cap +)}{P(+)} = \frac{(.2)(.85)}{(.2)(.85)+(.8)(.10)}$$

$$= .68$$

c) $$P(XYZ \mid + \text{ twice}) = \frac{(.2)(.85)(.85)}{(.2)(.85)(.85)+(.8)(.1)(.1)}$$

$$\approx .9475$$

$P(E|F) = \boxed{\dfrac{P(E\cap F)}{P(F)} = P(E)}$ if E and F are indep

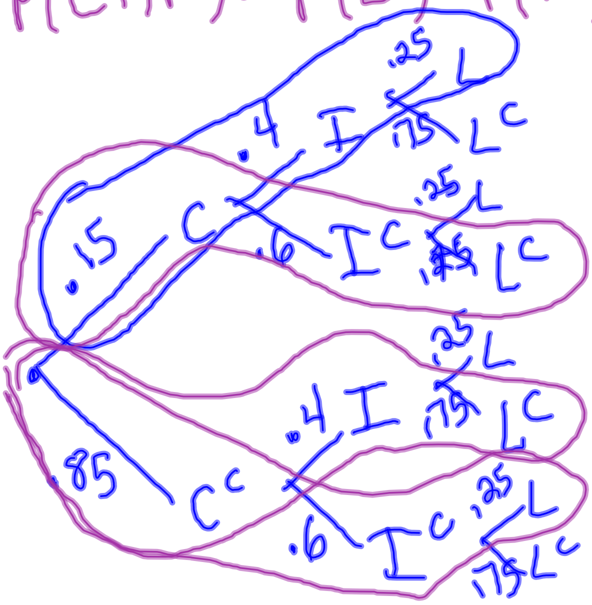
Example

Pyxie has a cell phone, an ipod and a laptop. Each morning Pyxie finds that the probability the cell phone battery is dead is 15%, the probability that the ipod battery is dead is 40% and the probability that the laptop battery is dead is 25%.

(a) What is the probability that all three have dead batteries? none, 1 dead, 2 dead, 3 dead

(b) What is the probability that exactly one of these three devices has a dead battery? *

$P(E\cap F) = P(E)\cdot P(F)$ iff E and F are indep

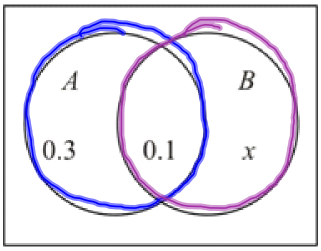


a) $P(C\cap I\cap L)$
$= (.15)(.4)(.25)$
$= 0.015$

$P(C\cap I^c\cap L^c) + P(C^c\cap I\cap L^c) + P(C^c\cap I^c\cap L)$
$= (.15)(.6)(.75) + (.85)(.4)(.75) + (.85)(.6)(.25)$
$= 0.45$

Example

For what value of x will A and B be independent?



$P(A \cap B) = P(A) \cdot P(B)$ if A and B are indep

$$P(A \cap B) = P(A) \cdot P(B)$$

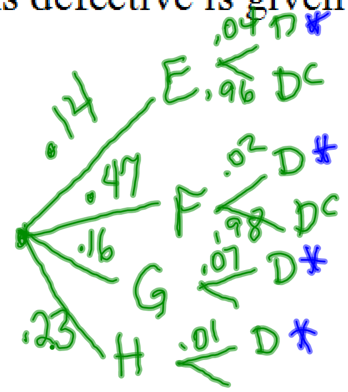
$$\frac{0.1}{.4} = \frac{\cancel{.4} \cdot (.1 + x)}{\cancel{.4}}$$

$$.25 = .1 + x \Rightarrow x = .15$$

Example

A manufacturer buys items from each of 4 different suppliers. The fraction of the total number of items obtained from each supplier, along with the probability that an item purchased from the supplier is defective is given in the following table:

supplier	fraction of total supplied	probability of defect
E	0.14	0.04
F	0.47	0.02
G	0.16	0.07
H	0.23	0.01



(a) What is the probability that an item is defective and from supplier H? $P(D \cap H) = (.23)(.01) = .023$

(b) What is the probability that a defective item came from

$$\boxed{P(F \mid D) = \frac{P(F \cap D)}{P(D)}}$$

$$= \frac{P(F \cap D)}{P(E \cap D) + P(F \cap D) + P(G \cap D) + P(H \cap D)}$$

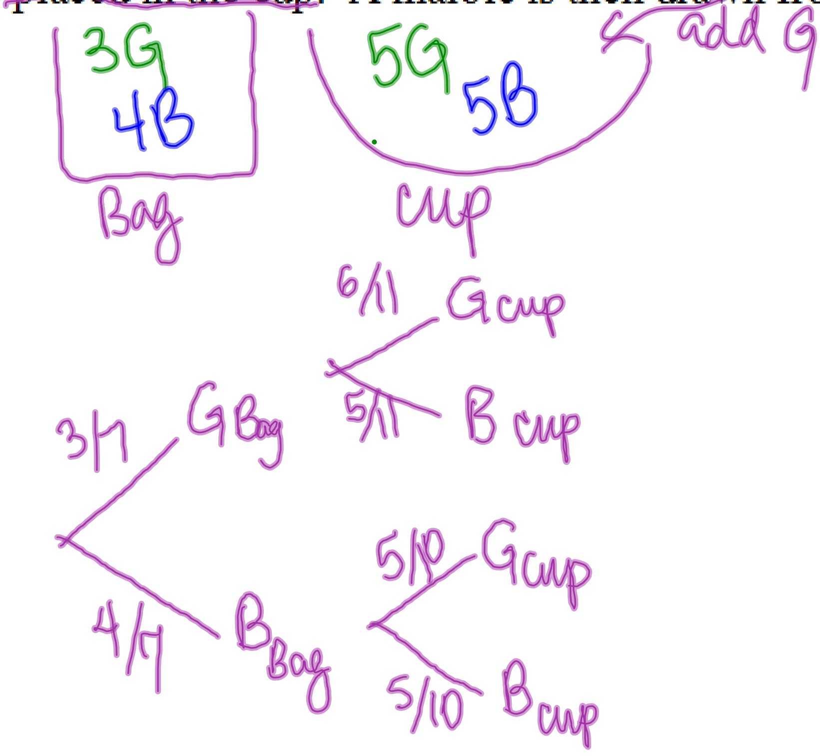
$$= \frac{(.47)(.02)}{(.14)(.04) + (.47)(.02) + (.16)(.07) + (.23)(.01)}$$

$$= \frac{94}{285} \approx 0.3298$$

Example

Draw the appropriate tree diagram for these experiments.

A bag has 3 green and 4 blue marbles. A cup has 5 green and 5 blue marbles. A marble is chosen at random from the bag. If it is blue, it is returned to the bag. If it is green, it is placed in the cup. A marble is then drawn from the cup.



We choose a marble from a cup or a bowl. The probability of choosing the cup is 1/4. The cup contains 3 purple and 2 yellow marbles. The bowl contains 2 purple and 4 yellow marbles.