

WEEK 14 REVIEW – Finance Part 2 and Markov Chains

AMORTIZATION

Example: A credit card charges 24% annual interest compounded monthly on the unpaid balance. You owe \$4000 on this credit card. To pay it off, you stop using it and make monthly payments of \$100.

- (a) How long until the card is paid off?
- (b) How much was paid in interest charges?
- (c) How much of the first payment was interest?
- (d) How much of the second payment was interest?
- (e) How much is still owed on the credit card after two years of making payments?

Example: A house costs \$180,000. You make a down payment of \$20,000 and finance the remainder for 20 years at 5.1% annual interest compounded monthly on the unpaid balance.

- (a) How large are the monthly payments?
- (b) How much interest is paid in all?
- (c) Fill in the blanks on the amortization table below:

end of period	remaining payments	payment	interest paid	towards principal	outstanding balance	equity
0	360	n/a	n/a	n/a		
1						
2						
12						
60						
120						
180						

MARKOV CHAINS

Example

Bob buys a cup of coffee or tea every day. If he buys a cup of coffee, there is a 30% chance he will buy a cup of coffee the next day and a 70% chance he will buy a cup of tea. If he buys a cup of tea, there is a 50% chance that he will buy a cup of tea the next day and a 50% chance he will buy a cup of coffee.

- (a) Is this a Markov process?
- (b) Find the transition matrix.
- (c) On his first day back from vacation, Bob buys a cup of coffee. What is the probability that he buys a cup of coffee 3 days later?
- (d) What are the long term (steady state) probabilities that Bob buys a cup of coffee or tea?

Example

In a certain city elections are held every two years for mayor. There are three political parties in this city, A, B and C. If the current mayor is from the A party, there is a 20% chance that the next mayor will be from the A party, 40% from the B party and 40% from the C party. If the current mayor is from the B party, there is a 50% chance the next mayor will be from the A party and a 50% chance that the mayor will be from the C party. If the current mayor is from the C party, there is a 60% chance the next mayor will be from the C party, a 10% chance the next mayor will be from the A party and a 30% chance the next mayor will be from the C party.

- (a) Is this a Markov process?
- (b) Find the transition matrix.
- (c) On the city's first election there is an equal chance that the mayor comes from parties A, B and C. What is the probability that the mayor is from party A in 8 years?
- (d) What is the long term (steady state) distribution of the mayor's political party?

Example

There are three brands of cell phones given to employees of a company. Each year the employee can choose phone brand X, phone brand Y or phone brand Z. If an employee has a brand X phone, he will choose a brand X phone again the next year. If an employee has a brand Y phone, there is an equal chance that he will choose brand X, Y or Z the next year. If a person has a brand Z phone, he will choose a brand X phone 50% of the time, a brand Y phone 25% of the time and a brand Z phone 25% of the time.

- (a) Is this a Markov process?
- (b) Find the transition matrix.
- (c) Initially all employees are given brand Z phones. What is the distribution of phone ownership in 2 years?
- (d) What is the long term (steady state) distribution of phone ownership?

Example

Classify the following matrices as a regular transition matrix, not a regular transition matrix, or not a transition matrix.

(a) $\begin{bmatrix} 0 & 0.4 \\ 1 & 0.6 \end{bmatrix}$

(b) $\begin{bmatrix} 0.5 & 0.8 \\ 0.5 & 0.6 \end{bmatrix}$

(c) $\begin{bmatrix} 0.75 & 0 \\ 0.25 & 1 \end{bmatrix}$