WEEK 2 REVIEW –

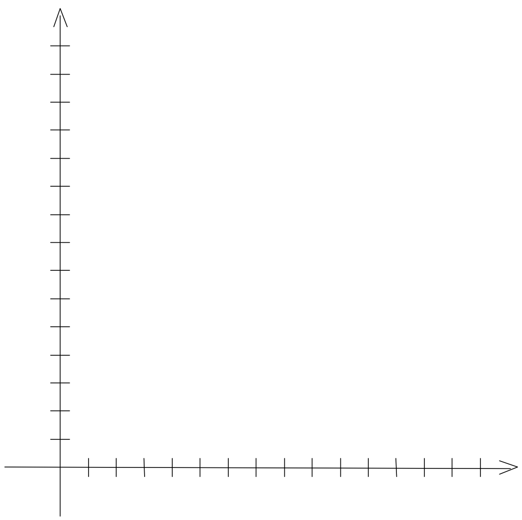
Linear Regression and Systems of Linear Equations

Example

The table below shows *x*, the number of cartons of blueberries, that a fruit stand can sell at different prices *y* in dollars.

x	8	11	20	28
y	6.9	5.5	4	3.35

(a) Show this data in a scatter diagram (plot)



(b) Use linear regression to find the best-fitting line for the price of blueberries.

Press the STAT button and then press ENTER to edit your lists	Enter the values for *x* in list L1 and *y* in list L2	Press 2^{ND} and QUIT to return to the homescreen. Press STAT and right arrow to CALC	Choose 4:LineReg(ax+b) and press ENTER:
EDIT CALC TESTS 1:Edit… 2:SortA(3:SortD(4:ClrList 5:SetUpEditor	L1	L2	L3 2 8	6.9 11	5.5 20	4 28	3.35 L2(1)=6.9	EDIT CALC TESTS 1:1-Var Stats 2:2-Var Stats 3:Med-Med 4:LinReg(ax+b) 5:QuadReg 6:CubicReg 7↓QuartReg	LinReg(ax+b)

Use L1 for x and L2 for y	Press ENTER to get the best-fitting line:	If r and r^2 are not displayed, go to CATALOG (2^{ND} and the 0 button) and choose DiagnosticON.	To graph the data and best-fitting line, press 2^{ND} and Y= to access the STATPLOT
LinReg(ax+b) L1, L2	LinReg y=ax+b a=-.1674265451 b=7.74189463 r²=.9140905782 r=-.9560808429	CATALOG Degree DelVar DependAsk DependAuto det(DiagnosticOff ▸DiagnosticOn	STAT PLOTS 1:Plot1…On L1 L2 2:Plot2…Off L1 L2 3:Plot3…Off L1 L2 4↓PlotsOff

Press ENTER to set-up Plot1:	To enter the best-fitting line, press Y= and then the VARS button:	Choose 5: Statistics and right arrow twice to EQ:	Press ENTER to paste RegEQ into Y1=:
Plot1 Plot2 Plot3 Off Type: Xlist:L1 Ylist:L2 Mark: ▫ + ·	VARS Y-VARS 1:Window… 2:Zoom… 3:GDB… 4:Picture… 5:Statistics… 6:Table… 7:String…	XY Σ EQ TEST PTS 1:RegEQ 2:a 3:b 4:c 5:d 6:e 7↓r	Plot1 Plot2 Plot3 \Y1=-.1674265450 8612X+7.74189463 01925 \Y2= \Y3= \Y4= \Y5=

(c) What is the selling price of blueberries when 35 cartons are sold?

(d) If the fruit stand charges $5.00 for a carton of blueberries, use the best-fitting line to estimate how many cartons will be sold.

(e) If the data in the table represented the number of cartons of blueberries in thousands sold by a large grocery chain, estimate how many cartons will be sold at a price of $3.50 per carton.

SYSTEMS OF LINEAR EQUATIONS

When you have a system of two equations and two unknowns we have three possibilities for the two lines:

The two lines intersect.
The solution is the single
point of intersection.

The two lines are the same.
The solution is the entire line.

The two lines are parallel.
No intersection therefore no solution.

Example:
Solve the following systems of linear equations.

(a)
$$\begin{aligned} x+2y&=12 \\ 2x+3y&=19 \end{aligned}$$

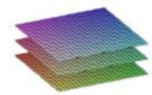
(b)
$$\begin{aligned} 2x-4y&=8 \\ -x+2y&=4 \end{aligned}$$

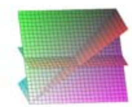
(c)
$$\begin{aligned} -x+3y&=7 \\ 2x-6y&=-14 \end{aligned}$$

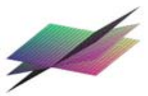
<u>Number of Solutions Theorem</u>

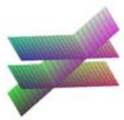
If the number of equations in a system of linear equations is equal to or greater than the number of variables, the system may have

- No solution
- Exactly one solution
- A parametric solution



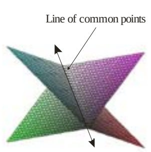






If the number of equations in a system of linear equation is less than the number of variables, then the system may have

- No solution
- A parametric solution



WORD PROBLEMS

Example:

Jane invests $10,000 in three ways. With one part she buys mutual funds with a return of 6.5% per year. The second part (which is twice as large as the 1^{st} part) is used to buy government bonds that pay 6% per year. The rest is put into a savings account paying 5% per year. In the 1^{st} year her average return was 6.05%.. How much did she invest in each way?

Example:

Farmer Blue has 100 acres available to plant white and yellow corn. Each acre of white corn will yield 95 bushels of corn and each acre of yellow corn will yield 120 bushels of corn. He wants to have at least three times as many bushels of white corn than he does of yellow corn. How many acres of each type of corn should Farmer Blue plant?

GAUSS-JORDAN

Example
Solve the following system of linear equations:

$$\begin{aligned} -3x - 6y + 6z &= -3 \\ 2x + 7y + 2z &= -1 \\ -x - 6y - 7z &= 3 \end{aligned}$$

1. Any two equations may be interchanged.
2. An equation may be multiplied by a non-zero constant.
3. A multiple of one equation may be added to another equation.

Augmented Matrix

A matrix is in Reduced-Row Echelon Form if
1. Each row consisting entirely of zeros lies below any row having non-zero entries.
2. The 1^{st} non-zero entry in any row is a 1 (called a leading 1)
3. In any two successive (non-zero) rows the leading 1 in the lower row lies to the right of the leading 1 in the upper row.
4. If a column contains a leading 1, the rest of the column is 0.

IMPORTANT!

- Only consider entries to the LEFT of the vertical line when Appling the definition of RREF.
- If a matrix is in RREF form, it may have one solution, no solution or a parametric solution.

$$\begin{aligned} -3x - 6y + 6z &= -3 \\ 2x + 7y + 2z &= -1 \\ -x - 6y - 7z &= 3 \end{aligned}$$

Example

Solve the following systems of linear equations:

(a) $$\begin{aligned} 2x + y - z &= 0 \\ 3x - y + 2z &= 1 \\ x - 2y + 3z &= 2 \end{aligned}$$

(b) $$\begin{aligned} 2x + y - 4z &= 10 \\ x + 2y + z &= 5 \\ x + y - z &= 5 \end{aligned}$$

Example

A zoo is looking to acquire some lions, tigers and bears. The zoo has 2800 square feet of space available and $850 for transportation costs. A lion needs 200 square feet of space and costs $50 to transport. A tiger needs 400 square feet of space and costs $150 to transport. A bear needs 400 square feet of space and costs $50 to transport. How many lions, tigers and bears can the zoo get?