Exam 3 - An Overview of what you need to know…

Chapter 7.2 – 7.4

- A sample space is uniform if every outcome has the same chance of occurring.
- The probability of event E in a uniform sample space S is $P(E) = \dfrac{n(E)}{n(S)}$.
- A simple event contains exactly one outcome. *
- $P(E \cup F) = P(E) + P(F) - P(E \cap F)$
- A probability distribution table has the following properties:
  - Each of the entries is mutually exclusive with all other entries
  - The sum of the probabilities is 1
- The empirical probability of an event is the relative frequency of the event

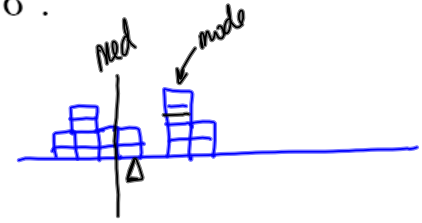
Chapter 7.5 – 7.6

- The conditional probability of event E given F is

  * $\boxed{P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}}$. *
- Condition probability can be found using trees, Venn diagrams, tables, etc.
- Product rule: $P(E \cap F) = P(F) \cdot P(E \mid F)$
- $\boxed{\text{Events } E \text{ and } F \text{ are independent if and only if } P(E \cap F) = P(F) \cdot P(E)}$

Chapter 8.1 – 8.3

- Random variables can be finite discrete, infinite discrete, or continuous
- Finite discrete random variables can be represented in a histogram.
- The expected value of a random variable X is given by $E(X) = x_1 \cdot p_1 + x_2 \cdot p_2 + \ldots x_n \cdot p_n$
- The odds in favor of an event E occurring is the ratio of $P(E)$ to $P(E^c)$ or $\frac{P(E)}{P(E^c)} = \frac{P(E)}{1-P(E)}$
- Given the odds in favor of an event E are a:b, the probability of E is given by $\frac{a}{a+b}$
- The mean of the n numbers $x_1, x_2, \ldots, x_n$ is $\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$
- The median of the n numbers $x_1, x_2, \ldots, x_n$ is the number in the middle when the n number are arranged in order of size and there are an odd number of values. When there is an even number of values, the median is the mean of the two middle numbers.
- The mode of the n numbers $x_1, x_2, \ldots, x_n$ is the number that occurs the most often. If no number occurs more often than any other number, there is no mode. If two numbers both occur the most often, then there are two modes.
- The standard deviation σ is a measure of how spread out the data is from the mean. Var = σ^2.



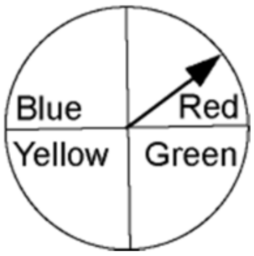
Chapter 8.4

- In a Bernoulli trial we have the following:
  - The same experiment repeated ~~several~~ a fixed # of times times.
  - The only possible outcomes of these experiments are success or failure.
  - The repeated trials are independent so the probability of success remains the same for each trial.
- Calculator commands are
  binompdf(n, p, x) and binomcdf(n, p, x)
- Mean is $\mu = Np$ and standard deviation is $\sigma = \sqrt{Np(1-p)}$

Part I – Basic Probability

1. Find the uniform sample space for the following experiments:

(a) A spinner is marked equally with the colors blue, red, yellow and green as shown. The spinner is spun and the color noted (if the needle lands on the line, it is spun again) and a fair coin is tossed.



B < H, T
R <
G <
Y <

$S = \{BH, BT, RH, RT, GH, GT, YH, YT\}$

$\underset{\text{spin}}{\underline{4}} \cdot \underset{\text{coin}}{\underline{2}} = 8$

$S = \{RR, RG, GR, GG\}$ NOT UNIFORM

$S = \{RR, RG, GG\}$ NOT UNIFORM

(b) A bag has 2 red and 3 green apples. A sample of two is chosen at random.

$C(5,2) = 10$

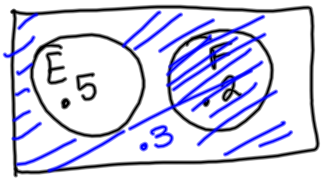
same

R_1 — R_2, G_1, G_2, G_3

R_2 — R_1

$\Rightarrow P(E \cap F) = 0$

2. Let E and F be two mutually exclusive events. Suppose that P(E) = 0.5 and P(F) = 0.2. Find $P\left(E^c \cup F\right)$ $= .3 + .2 = .5$



3. A pair of fair six-sided dice (one red and one green) is rolled. What is the probability that the red die shows a 3 or the sum of the numbers shown is less than 5?

E F

F

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

$\frac{11}{36}$

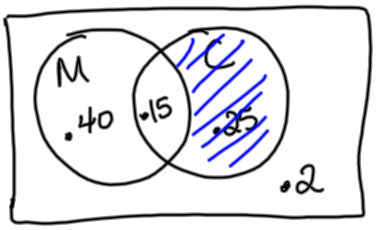
$P(E) = 6/36$

$P(F) = 6/36$

$P(E \cap F) = 1/36$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$
$= 6/36 + 6/36 - 1/36 = 11/36$

4. A group of students is surveyed and 55% of the group is men and 40% of the group like coffee. If 80% of the group are men or like coffee, find the probability that a student is a woman who likes coffee.



$P(M) = .55$

$P(C) = .40$

$P(M \cup C) = .80 = \overset{.55}{\cancel{P(M)}} + \overset{.4}{\cancel{P(C)}} - P(M \cap C)$

$\rightsquigarrow P(M \cap C) = .15$

$P(M^c \cap C) = .25$

5. A buffet has slices of pepperoni pizza on it. The number of pieces of pepperoni on each slice is counted and the following results are found:

X	No. of pieces of pepperoni on a slice	3	4	5	6	7
FREQ	No. of slices	1	4	9	6	2
PROBS		1/22	4/22	9/22	6/22	2/22

} 22 slices

What is the probability that a randomly selected slice of pizza will have more than 5 pieces of pepperoni?

8/22

6. Organize the following information into a probability distribution table: The tomatoes in a large box of tomatoes are weighed and the following results are found: 10% of the tomatoes weigh less than 4 ounces, 30% of the tomatoes weigh 8 or fewer ounces and 15% of the tomatoes weigh more than 12 ounces.

x = wt of a tomato in oz

Event	Prob
$x < 4$	.10
$4 \le x \le 8$	.20
$8 < x \le 12$	0.55
$x > 12$	.15

$\underbrace{.10 + .20}_{.30}$

$1 - .1 - .2 - .15$

7. A stack of 100 copies has 8 defective copies in it. A sample of 10 is chosen. What is the probability that the sample will have no defective copies?

Def Good

$$P = \frac{C(8,0)\,C(92,10)}{C(100,10)} = .4166$$

$= 8nCr0 * 92nCr10 / 100nCr10$

8. A bowl has 6 green, 7 red and 4 purple jelly beans. A sample of 4 is chosen at random. What is the probability that the sample will have exactly 3 green or exactly one purple jelly bean?

E F

6nCr3 * 11nCr1

$n(E) = \underbrace{C(6,3)\,C(11,1)}_{3G \text{ and } 1G^c} = 220$

$n(S) = C(17,4) = 2380$

$n(F) = \underbrace{C(4,1)\cdot C(13,3)}_{1P \text{ and } 3P^c} = 1144$

$n(E \cap F) = \underbrace{C(6,3)\,C(4,1)\,C(7,0)}_{3G \text{ and } 1P} = 80$

$P(E \cup F) = \dfrac{220 + 1144 - 80}{2380} = \dfrac{1284}{2380} \approx 53.9\%$

9. Four couples go to the movies. If all 8 people sit down randomly, what is the probability that couples are seated together?

$n(S) = \underline{8}\cdot\underline{7}\cdot\underline{6}\cdot\underline{5}\cdot\underline{4}\cdot\underline{3}\cdot\underline{2}\cdot\underline{1} = 40{,}320$

$n(E) = \underbrace{4!}_{\text{arr couples}} \cdot \underbrace{2!}_{C1} \cdot \underbrace{2!}_{C2} \cdot \underbrace{2!}_{C3} \cdot \underbrace{2!}_{C4}$

$C_1 \quad C_2 \quad C_3 \quad C_4$

$= 384$

$P(E) = \dfrac{384}{40320} = \dfrac{1}{105} \approx 0.0095$

Part II – Conditional Probability

1. Given $P(E) = 0.4$, $P(F) = 0.2$ and $P(E \cup F) = 0.5$,
   (a) Are E and F independent?
   (b) Are E and F mutually exclusive? NO B/c $P(E \cap F) \neq 0$

a) $P(E \cap F)$? $P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$.5 = .4 + .2 - P(E \cap F)$

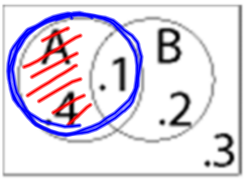
$\Rightarrow P(E \cap F) = .1$

$P(E) \cdot P(F) = .4 * .2 = 0.08 \neq P(E \cap F) = .1$

$\Rightarrow$ NOT INDEP

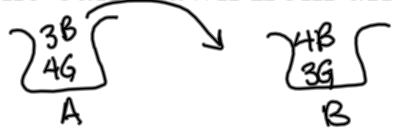
GIVEN A

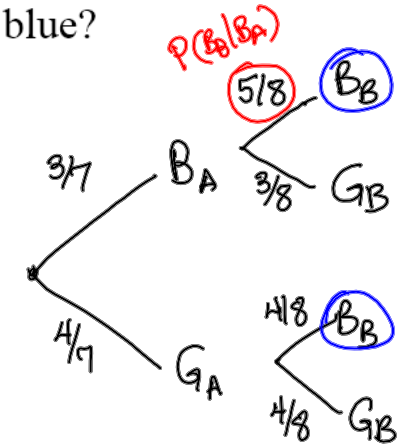
2. Find $P(B^C | A)$ from the Venn diagram:



$$\frac{P(B^c \cap A)}{P(A)} = \frac{.4}{.5} = \frac{4}{5}$$

3. Urn A has 3 blue and 4 green balls. Urn B has 4 blue and 3 green balls. A ball is chosen from urn A and placed in urn B. A ball is then chosen from urn B. What is the probability that the transferred ball was blue given that the ball drawn from urn B is blue?

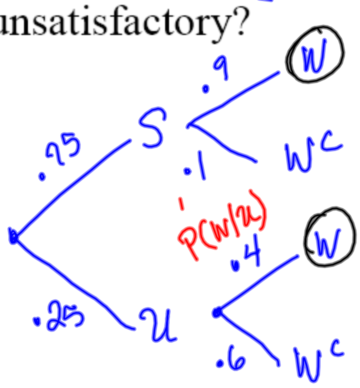




$$P(B_A|B_B) = \frac{P(B_A \cap B_B)}{P(B_B)}$$

$$= \frac{(3/7)(5/8)}{(3/7)(5/8) + (4/7)(4/8)} = \frac{15}{31} \approx .4838$$

4. A company has rated 75% of its employees as satisfactory and 25% is unsatisfactory. Personnel records indicate that 90% of those rated satisfactory had previous work experience and 40% of those rated unsatisfactory had previous work experience. What is the probability that an employee with previous work experience is unsatisfactory?

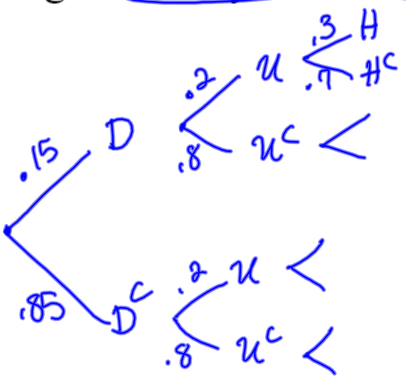


$$P(U|W) = \frac{P(U \cap W)}{P(W)}$$

$$= \frac{(.25)(.4)}{(.75)(.9) + (.25)(.4)} = \frac{4}{31} \approx .1290$$

5. A medicine has 3 common side effects. The probability a person taking this medicine gets drowsy is 15%, the probability a person taking this medicine gets an upset stomach is 20%, and the probability a person taking this medicine gets a headache is 30%

(a) What is the probability that a person taking this medicine gets all of these side effects?
(b) What is the probability that a person taking this medicine gets none of these side effects?
(c) What is the probability that a person taking this medicine gets exactly one of these side effects?



a) $P(D\cap U\cap H) = (.15)(.2)(.3) =$

b) $P(D^c\cap U^c\cap H^c) = (.85)(.8)(.7) =$

c) $P(D\cap U^c\cap H^c) + P(D^c\cap U\cap H^c)$
$+ P(D^c\cap U^c\cap H)$
$= (.15)(.8)(.7) + (.85)(.2)(.7)$
$+ (.85)(.8)(.3)$

6. Two fair six-sided dice are rolled. Given that the sum shown uppermost is five, what is the probability that a 3 is shown on one of the two dice?

$P = \frac{2}{4}$

1~1	2~1	3~1	4~1	5~1	6~1
1~2	2~2	3~2	4~2	5~2	6~2
1~3	2~3	3~3	4~3	5~3	6~3
1~4	2~4	3~4	4~4	5~4	6~4
1~5	2~5	3~5	4~5	5~5	6~5
1~6	2~6	3~6	4~6	5~6	6~6

exactly one 6

two sixes

ex one 6

7. Two cards are chosen in succession from a standard deck of 52 cards. Given that the second card is a heart, what is the probability that the first card was a diamond?

Part III – Random Variables and Statistics

1. A sample of jelly bean bags is chosen and the number of blue jelly beans in each bag is counted. The results are shown in the table below: PROB

	10/40	9/40	8/40	7/40	6/40	
L2 ← FREQ No. of bags	10	9	8	7	6	⇒ 40 Bags
L1 ← X No. of blue jelly beans	8	9	10	11	12	

(a) What is the expected number of blue jelly beans?

(b) What is the mean, median, mode, and standard deviation in the number of jelly beans?

a) $E = 8(10/40) + 9(9/40) + 10(8/40) + 11(7/40) + 12(6/40) = 9.75$

b) 1-var stats $L_1, L_2 \Rightarrow \bar{x} = 9.75$, med $= 10$, $\sigma = 1.3919$

mode is 8

2 3 | 4 5

$\frac{3+4}{2} = 3.5$ MED

2 3 (4) 5 6

2. A bag contains 10 oranges and 2 of them are rotten. What is the expected number of rotten oranges in a sample of 2?

outcome	X	P(X)
2 rotten	2	$\frac{C(2,2)C(8,0)}{C(10,2)} = 1/45$
1 rotten	1	$\frac{C(2,1)C(8,1)}{C(10,2)} = 16/45$
0 rotten	0	$\frac{C(2,0)C(8,2)}{C(10,2)} = 28/45$
	L1	L2

$E = 2(1/45) + 1(16/45) + 0(28/45)$

$= 0.4$

0, 1, 1, 0, 2, 0, 0, 0 . . .

$\Rightarrow$ 1-var stats $L_1, L_2 \Rightarrow \bar{x} = 0.4$

3. Find the range of values for the random variable X in the following experiments and determine if the random variable is finite discrete, infinite discrete or continuous.
(a) Let X be the number of queens in a hand of 5 cards.
(b) Let X be the time in seconds to swim a 50m race
(c) A bowl has 5 red and 5 green marbles. One marble is chosen at random. If the marble is green, it is replaced in the bowl. Let X be the number of times a marble is chosen until a red marble is picked.

(a) $X = 0, 1, 2, 3, 4$ Finite discr r.v.

(b) $X \geq 0$ Continuous

(c) $X = 1, 2, \ldots$ Infin. disc r.v.

4. A game is played where a person pays to roll two fair six-sided dice. If exactly one six is shown uppermost, the player wins $5. If exactly 2 sixes are shown uppermost, then the player wins $20. How much should be charged to play this game is the player is to break-even? Round to the nearest cent.

outcome	X	P(X)
one 6	5	10/36
two 6's	20	1/36
zero 6's	0	25/36

$E = 5\left(\frac{10}{36}\right) + 20\left(\frac{1}{36}\right) + 0\left(\frac{25}{36}\right) - C$

COST TO PLAY

$= 0$

$\to C = \$1.94$

5. Mr. Smith buys a $4000 insurance policy on his son's violin. The premium is $50 per year. If the probability that the violin will need to be replaced is 0.8%, what is the insurance company's gain (if any) on this policy?

$\frac{.8}{100}$

outcome	X	PROB
replace	50 − 4000	0.008
not replace	50	0.992

$E = (-3950)(.008) + 50(.992)$

$= \$18$

6. The odds in favor that a horse will win a race are 3:11. What is the probability the horse will win?

$\frac{3}{14}$

7. The probability of rain is 60%. What are the odds in favor of rain?

$\frac{P(E)}{P(E^c)} = \frac{.6}{1-.6} = \frac{.6}{.4} \Rightarrow 3:2$ or 3 to 2

8. The following data is the recorded daily high temperature in College Station for March 2006:

L1 {83, 81, 77, 74, 77, 83, 80, 82, 79, 85, 86, 86, 75, 72, 69, 77, 72, 69, 76, 76, 65, 58, 51, 61, 69, 74, 72, 67, 73, 81, 82

Find the mean, median, mode and standard deviation for the daily high temperature.

1 var stats L1 $\Rightarrow$ mean 74.58, med 76, $\sigma = 8.1430$
mode 69, 72, 77

Part IV – Binomial Probability

$p = 0.002$

1. The probability that a transistor is defective is 0.2%. A box contains 12 transistors. What is the probability that a box contains at least one defective transistor?

3. A basketball player has an 80% chance of making a free throw. If she has 10 attempts in a game, what is the probability that she make 4 of her first 6 attempts and 3 of her last 4 attempts?

$n=100$

2. At a local restaurant 100 people ate bad tuna salad. The probability of getting food poisoning from bad tuna salad is 40%. $p=.4$

(a) What is the probability that fewer than 30 people get sick?

(b) What is the probability that 45 or more people get sick?

(c) What is the probability that between 40 and 50 people get sick?

(d) What is the expected number of sick people?

(e) What is the standard deviation in the number of people who get sick?

(a) $X=0,1,2,\ldots,29$

(b) $X=45,46,\ldots,100$

(c) $X=41,42,\ldots,49$

$\mu = 100 * .4$

$\sigma = \sqrt{100(.4)(.6)}$