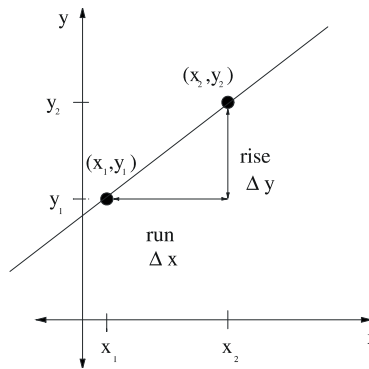


WEEK 1 REVIEW – Lines and Linear Models

SLOPE

A VERTICAL line has NO SLOPE. All other lines have

$$\text{slope} = m = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$



Example

Find the slope of the line passing through the points $(-2, 4)$ and $(0, -4)$

Answer

Let one pair of points be (x_1, y_1) and the other (x_2, y_2) . Then

If we assigned our points the other way we would have

EQUATIONS OF LINES

The formula for the slope of a line can be rearranged to give us the equation for a line.

$$m = \frac{y - y_1}{x - x_1} \rightarrow y - y_1 = m(x - x_1)$$

This is called the POINT-SLOPE form of a line. If you know a point, (x_1, y_1) that lies on the line and you know the slope, m , of the line, then you can find the equation of the line.

Example

What is the equation of the line passing through the points $(-2, 4)$ and $(0, -4)$?

Answer

$m = -4$ (previous example) Let $(x_1, y_1) = (-2, 4)$

Let $(x_1, y_1) = (0, -4)$

When we simplify our point-slope form we are writing the line in the slope-intercept form,

$$y = mx + b$$

Again, m is the slope and now b is the y -intercept.

The y -intercept is the place where the line crosses the y -axis.
The x -intercept is the place where the line crosses the x -axis.

Example

Graph the line $y = -4x - 4$ and find the intercepts.

Answer

$Ax + By = C$ is the GENERAL FORM of a line.

Example

Graph the line $3x - 4y = 12$ on paper and on the calculator.

Answer

Two lines are parallel if they have the same slope and different y-intercepts, $m_1 = m_2$ and $b_1 \neq b_2$

Two line are perpendicular if the product of their slopes is -1,

$$m_1 \cdot m_2 = -1 \text{ or } m_1 = \frac{-1}{m_2}$$

Example

Given the line L_1 is $y = 2x + 4$,

(a) find a line parallel to L_1 that passes through the point (4, 4)

(b) find a line perpendicular to L_1 that passes through the point (4,4)

Answer

APPLICATIONS

Example

In 2010 for wages less than the maximum taxable wage base, Social Security contributions by employees are 6.2% of the employee's wages.

- a) Find a linear model that expresses the relationship between wages and Social Security contributions for employees earning less than the maximum (\$106,800 in 2010).
- b) Graph this equation and find the social security contribution for an employee earning \$35,000 in wages in a year.

Answer

LINEAR BUSINESS MODELS

Depreciation: the value, V , of an item decreases linearly with time. The item has an initial value and then the value decreases by the same amount each time period.

Cost: in a linear cost model the TOTAL cost to make x items is $C(x) = cx + F$. F represents the *fixed costs*. These are the costs you have even if you make no items. c is the cost to make each unit, called the *variable cost*.

Revenue: in a linear revenue model the revenue from selling x items is $R(x) = sx$. s is the sale price of a single item.

Profit: the difference between the money in (revenue) and the money spent (costs) is the profit. $P(x) = R(x) - C(x)$

Supply: in a linear supply model the number of items, x , that a company will supply at a price p is given by $S(x) = p = m_s x + b_s$.

Demand: in a linear demand model the number of items, x , that consumers will purchase at a price p is given by $D(x) = p = m_D x + b_D$.

DEPRECIATION

Example

A car is purchased for \$18,000 and is kept for 7 years. At the end of 7 years the car is sold for \$4000. Find an equation that models the decrease in the value of the car over time. What is the car worth after 3 years?

Answer

COST, REVENUE and PROFIT

Example

Suppose a company manufactures baseball caps. In a day they can produce 100 caps for a total cost of \$600. If no caps are produced their costs are \$200 per day. The caps sell for \$8 each. Find the cost, revenue and profit equations.

Answer

SUPPLY AND DEMAND

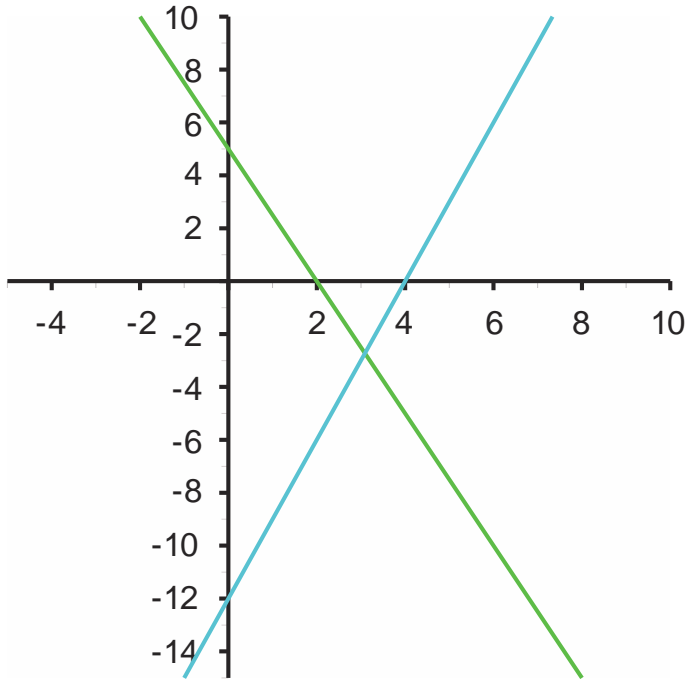
Example

A baker is willing to supply 16 jumbo cinnamon rolls to a café at a price of \$1.70 each. If she is offered \$1.50 for each roll, she will supply 4 fewer rolls to the café. At the café, customers will purchase no cinnamon rolls if the cost is \$7.20 each. However, if the price of a cinnamon roll is \$0.80, the café can sell 40 of these rolls.

Find the supply and demand equations for jumbo cinnamon rolls.

THE INTERSECTION OF TWO LINES

Find where the lines $10x + 4y = 20$ and $3x - y = 12$ intersect.



Break-even Point: This is where the cost to produce x items is the same as the revenue brought in from selling these x items. This occurs when $R(x) = C(x)$.

Example

Find and interpret the break-even point for making and selling baseball caps.

Equilibrium Point: This is the price p that the consumer and producer are willing to pay/accept for x items. This occurs when $S(x) = D(x)$

Example

Find and interpret the equilibrium point for the supply and demand for jumbo cinnamon rolls.

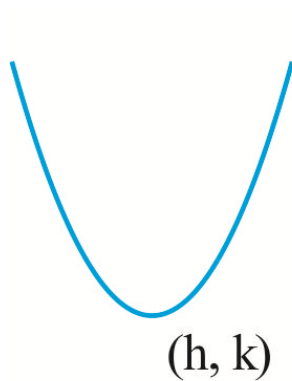
QUADRATICS

A quadratic is a polynomial of order 2:

$$y = ax^2 + bx + c, a \neq 0.$$

Every quadratic function can also be written in standard form:

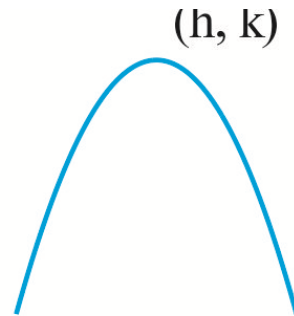
$$y = a(x - h)^2 + k \text{ where } h = -\frac{b}{2a} \text{ and } k = c - \frac{b^2}{4a}$$



$$a > 0$$

Concave up

Note the vertex is a min



$$a < 0$$

Concave down

Note the vertex is a max

The x-intercepts can be found using the quadratic formula:

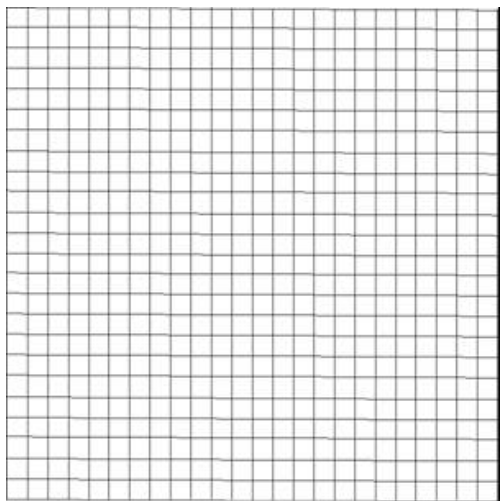
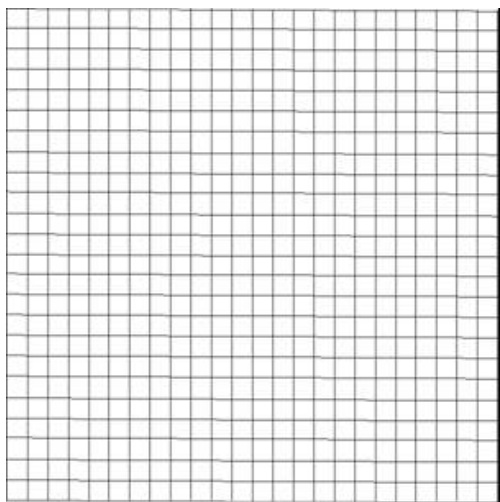
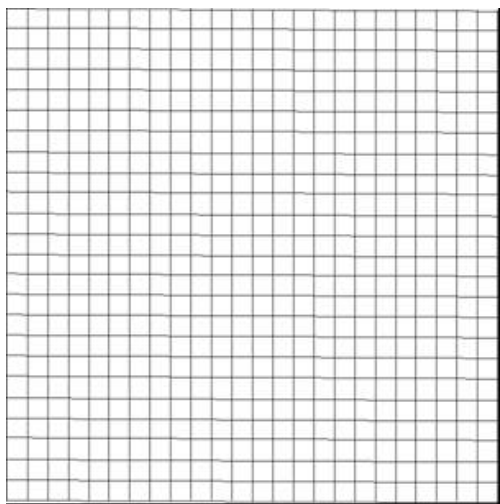
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ when } b^2 - 4ac \geq 0$$

Graph the following quadratics and find the intercepts and vertices.

$$y = x^2 + x - 12$$

$$y = 4x^2 + 16x + 2$$

$$y = -x^2 + 4x - 5$$



Example: What is the revenue from selling espressos if the demand equation for selling espressos is $p = -0.25x + 5$? Graph the revenue equation and interpret the result.

