WEEK 11 REVIEW (7.5 – 7.6 and 8.1 – 8.2)

Conditional Probability (7.5 – 7.6)

Notation: $P(E \mid F)$ is the probability of event E occurring given that event F has occurred.

Remember that $P(E \cap F) = \dfrac{n(E \cap F)}{n(S)}$

The ***conditional probability*** of event E given event F is

$$* \boxed{P(E \mid F) = \frac{n(E \cap F)}{n(F)}} = \frac{n(E \cap F)/n(S)}{n(F)/n(S)} = \frac{P(E \cap F)}{P(F)}$$

Since $* \boxed{P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}} *$, multiply both sides by $P(F)$ to get

Product rule: $\boxed{P(E \cap F) = P(F)P(E \mid F)}$

Independent events, that is, $P(E \mid F) = P(E)$, so if

$$P(E \mid F) = \frac{P(E \cap F)}{P(F)} = P(E) \quad \Rightarrow$$

$$* \boxed{P(E \cap F) = P(E) \cdot P(F) \textit{ iff } E \text{ and } F \text{ are independent}} *$$

Bayes' Theorem: $P(E \mid F) = \dfrac{P(E \cap F)}{P(F)}$

Example: A bowl has 15 pieces of fruit and 5 of the pieces of fruit are rotten. There are 9 apples (3 are rotten) and 6 oranges.
(a) What is the probability that a randomly selected piece of fruit is a rotten orange?
(b) What is the probability that an apple is rotten?
(c) What is the probability that a good piece of fruit is an orange?

	Rotten	Good	TOT
A	3	6	9
O	2	4	6
TOT	5	10	15

(a) $P(R \cap O) = \frac{n(R \cap O)}{n(S)} = \frac{2}{15}$

(b) $P(R \mid A) = \frac{n(R \cap A)}{n(A)} = \frac{3}{9}$

(c) $P(O \mid G) = \frac{n(O \cap G)}{n(G)} = \frac{4}{10}$

Example: Two fair five-sided dice are rolled.
(a) What is the probability that the sum is greater than 6? $\frac{10}{25} = P(>6)$

1~1	2~1	3~1	4~1	5~1
1~2	2~2	3~2	4~2	5~2
1~3	2~3	3~3	4~3	5~3
1~4	2~4	3~4	4~4	5~4
1~5	2~5	3~5	4~5	5~5

(b) What is the probability that the sum is greater than 6 if at least one 3 is shown? ?

$P(>6 \mid \text{at least 1 3 is showing}) = \frac{4}{9}$

(c) What is the probability that the sum is 4 if both dice display the same number?

1~1	2~1	3~1	4~1	5~1
1~2	2~2	3~2	4~2	5~2
1~3	2~3	3~3	4~3	5~3
1~4	2~4	3~4	4~4	5~4
1~5	2~5	3~5	4~5	5~5

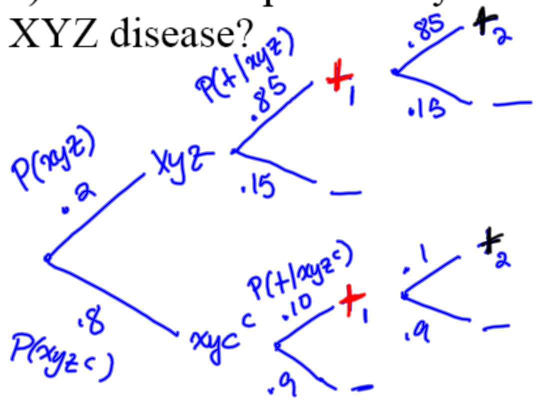
$\frac{1}{5}$

Example: A local clinic tests for XYZ disease. It is known that 20% of the patients coming to the clinic have XYZ disease. The test for XYZ is positive for 85% of the patients that have XYZ and it is positive for 10% of the patients that do not have this disease.

a) Represent this experiment in a tree diagram.

b) What is the probability that a person who tests positive has XYZ disease?

c) What is the probability that a person who tests positive twice has XYZ disease?



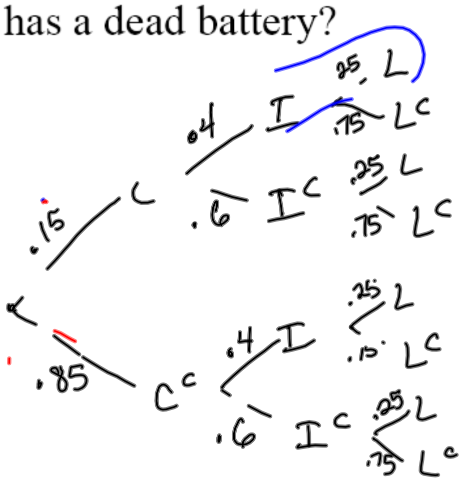
(b) $P(xyz \mid +) = \dfrac{P(xyz \cap +)}{P(+)} = \dfrac{(.2)(.85)}{(.2)(.85)+(.8)(.1)} = 0.68$

(c) $P(xyz \mid +_1 \cap +_2) = \dfrac{(.2)(.85)(.85)}{(.2)(.85)(.85)+(.8)(.1)(.1)} = .9475$

Example: Pyxie has a cell phone, an ipad and a laptop. Each morning Pyxie finds that the probability the cell phone battery is dead is 15%, the probability that the ipad battery is dead is 40% and the probability that the laptop battery is dead is 25%.

(a) What is the probability that all three have dead batteries?

(b) What is the probability that exactly one of these three devices has a dead battery?



(a) $P(\overset{\text{dead}}{C} \cap I \cap L) = (.15)(.4)(.25) = 0.015$

$= 1.5\%$

(b) $P(\overset{\text{dead}}{C} \cap I^c \cap L^c) + P(C^c \cap \overset{\text{dead}}{I} \cap L^c)$

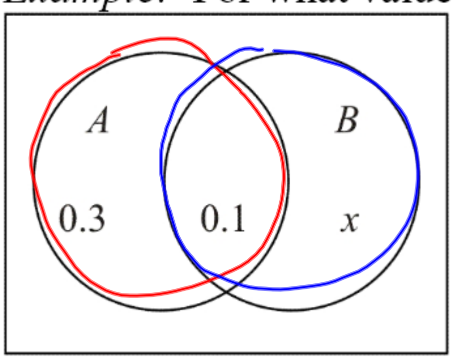
$+ P(C^c \cap I^c \cap \overset{\text{dead}}{L})$

$= (.15)(.6)(.75) + (.85)(.4)(.75)$

$+ (.85)(.6)(.25)$

$= 0.45$

Example: For what value of x will A and B be independent?



$\boxed{P(A) \cdot P(B) = P(A \cap B) \text{ iff } A \& B \text{ are indep}}$

$(.4) * (x + .1) = 0.1$

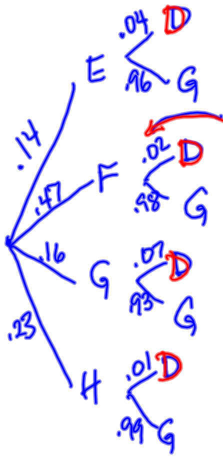
$.4x + .04 = 0.1$

$.4x = 0.06$

$x = \dfrac{0.06}{.4} = 0.15$

Example: A manufacturer buys items from each of 4 different suppliers. The fraction of the total number of items obtained from each supplier, along with the probability that an item purchased from the supplier is defective is given in the following table:

supplier	fraction of total supplied	probability of defect
E	0.14	0.04
F	0.47	0.02
G	0.16	0.07
H	0.23	0.01



$$P(D|F) = \frac{P(D \cap F)}{P(F)} = \frac{(\cancel{.47})(.02)}{(\cancel{.47})}$$

(a) What is the probability that an item is defective and from supplier H? $P(H \cap D) = (.23)(.01) = 0.0023$

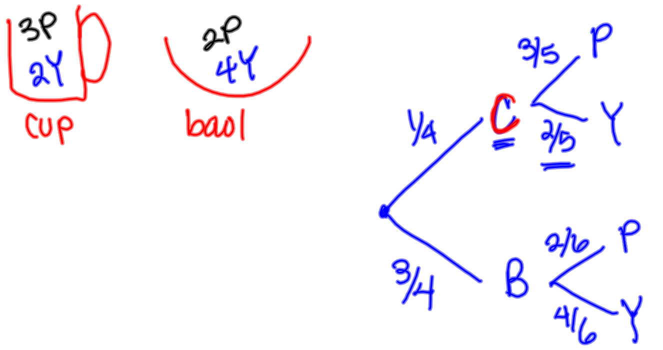
(b) What is the probability that a defective item came from supplier F?

$$P(F|D) = \frac{P(F \cap D)}{P(D)} = \frac{(.47)(.02)}{(.14)(.04) + (.47)(.02) + (.16)(.07) + (.23)(.01)}$$

$$= \frac{94}{285} \approx 0.3298$$

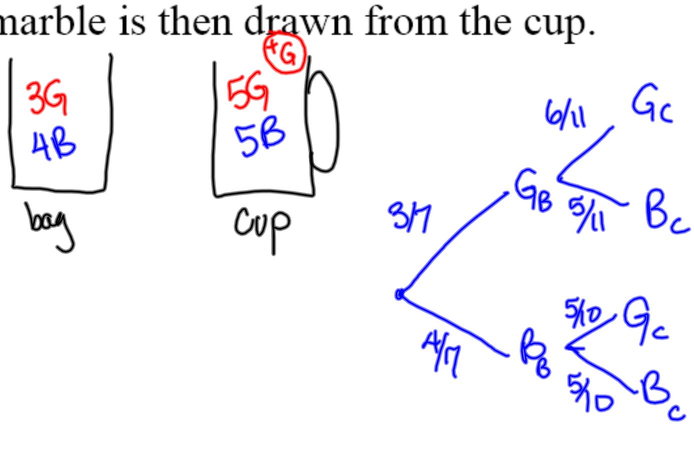
Example: Draw the appropriate tree diagram for the experiments.

We choose a marble from a cup or a bowl. The probability of choosing the cup is 1/4. The cup contains 3 purple and 2 yellow marbles. The bowl contains 2 purple and 4 yellow marbles.



$P(Y|C)$

A bag has 3 green and 4 blue marbles. A cup has 5 green and 5 blue marbles. 1 A marble is chosen at random from the bag. 2 If it is blue, it is returned to the bag. 3 If it is green, it is placed in the cup. A marble is then drawn from the cup.



Statistics (8.1 – 8.2)

Example: Determine the possible values for the given random variable and indicate if the random variable is finite discrete, infinite discrete or continuous.

(a) A hand of 5 cards is dealt from a standard deck of 52 cards. Let X be the number of clubs in the hand of cards.

$X = 0, 1, 2, 3, 4, 5$

Finite discrete r.v.

(b) A kitten is weighed. Let X be the weight of the kitten in pounds.

$X \geq 0$

continuous

(mass, length, time, temp)

(c) A single card is drawn without replacement from a standard deck of 52 cards. Let X be the number of cards drawn until a red card is picked.

$X = 1, 2, 3, 4, \ldots, 26, 27$

finite dis r.v.

(d) A single card is drawn with replacement from a standard deck of 52 cards. Let X be the number of cards drawn until a red card is picked.

infinite disc. r.v.

$X = 1, 2, 3, \ldots$

Example: Use the probability distribution table below to answer the following questions.

X	-2	-1	0	1	2	3
$P(X)$	0.15	0.05	0.2	0	z	0.25

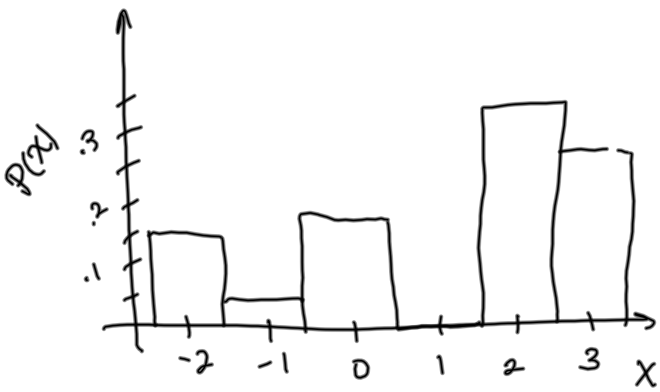
$\Rightarrow .65 + z = 1 \Rightarrow$

(a) Determine the value of z $z = .35$

(b) Represent this information in a histogram.

(c) $P(X \leq 0)$ $= .15 + .05 + .2 = .4$

(d) $P(X > 2)$ $= .25$



The expected value of a random variable X is given by

$$E(X) = x_1 \cdot p_1 + x_2 \cdot p_2 + \ldots x_n \cdot p_n$$

where x_i represents the values that X can have and p_i is the probability that x_i occurs

Example: A raffle sells 500 tickets at \$5 each. There is one first place prize awarded for \$1000, two second place prizes for \$100 each and 10 third place prizes at \$20 each. What is the expected value of a ticket in this raffle?

OUTCOME	X (net)	PROB
1st	1000 − 5	1/500
2nd	100 − 5	2/500
3rd	20 − 5	10/500
nothing	0 − 5	487/500

$$E = (995)(1/500) + (95)(2/500) + (15)(10/500) + (-5)(487/500)$$

$$= -2.20$$

$-\$2.20$

$$E = (1000)(1/500) + 100\,(2/500) + 20\,(10/500) + 0\,(487/500) - \underbrace{5}_{\text{COST TO PLAY}}$$

Example: An Aggie ring is insured for \$1200. The annual premium on the ring insurance is \$15 and the probability that the ring will need to be replaced is 0.5% What is the insurance company's expected gain on this policy?

Outcome	X	P(X)
replace	15 − 1200	0.005
not replace	15	0.995

$$E = (-1185)(.005) + (15)(.995) = 9$$

\$9

Example: A game consists of choosing two bills at random from a bag that contains two \$20 bills and ten \$1 bills. How much should be charged to play this game to keep it "fair" (expected value of zero)?

outcome	X	Prob
2 twenties	40	$C(2,2)C(10,0)/C(12,2) = 1/66$
1 twenty 1 one	21	$C(2,1)C(10,1)/C(12,2) = 20/66$
2 ones	2	$C(2,0)C(10,2)/C(12,2) = 45/66$

$$E = 40\left(\tfrac{1}{66}\right) + 21\left(\tfrac{20}{66}\right) + 2\left(\tfrac{45}{66}\right) - \underset{\text{COST TO PLAY}}{\underline{P}} = 0$$

$$\Rightarrow \boxed{p = \$8.33}$$

Example: A hand of 3 cards is dealt from a standard deck of 52 cards. Let *X* be the number of clubs in the hand of cards. Find the expected number of clubs

Event	X	Prob
$0C\ 3C^c$	0	$\frac{C(13,0)C(39,3)}{C(52,3)} = \frac{9139}{22100}$
$1C\ 2C^c$	1	$\frac{C(13,1)C(39,2)}{C(52,3)} = \frac{9633}{22100}$
$2C\ 1C^c$	2	$\frac{C(13,2)C(39,1)}{C(52,3)}$
$3C\ 0C^c$	3	$C(13,3)C(39,0)$

$$E = 0\left(\frac{9139}{22100}\right) + 1\left(\frac{9633}{22100}\right) + 2\left(\frac{3042}{22100}\right) + 3\left(\frac{286}{22100}\right) = .75$$

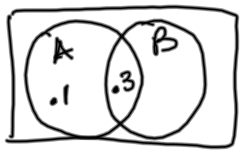
The **odds in favor** of an event E occurring is the ratio of $P(E)$ to $P(E^c)$ or $\dfrac{P(E)}{P(E^c)} = \dfrac{P(E)}{1-P(E)}$

Given the odds in favor of an event E are $a{:}b$, the probability of E is given by $\dfrac{a}{a+b}$

Example: The odds in favor of selected horses in the 2009 Kentucky Derby were
Backtalk 1:14 Homeboykris 1:60 Sidney's Candy 2:15

(a) What is the probability that Backtalk will win the race?
(b) What is the probability that Homeboykris will win the race?
(c) What is the probability that Sidney's Candy will win the race?

(a) $P = \frac{1}{1+14} = \frac{1}{15}$ (b) $P = \frac{1}{1+60} = \frac{1}{61}$ (c) $P = \frac{2}{2+15}$



Example: Given that $P(A)=0.4$, $P(B)=0.6$, and $P(A\cap B)=0.3$, find the odds in favor of
(a) A occurring (b) only A occurring

$\frac{P(A)}{P(A^c)} = \frac{.4}{1-.4} = \frac{.4}{.6} = \frac{2}{3} \Rightarrow 2:3$ odds

$P(A\cap B^c) = .1 \Rightarrow \frac{.1}{1-.1} = \frac{.1}{.9} = \frac{1}{9} \Rightarrow 1:9$ odds

Measures of Central Tendency

The **mean** of the n numbers $x_1, x_2, \ldots, x_n$ is $\bar{x} = \dfrac{x_1 + x_2 + \cdots + x_n}{n}$

The **median** of the n numbers $x_1, x_2, \ldots, x_n$ is the number in the middle when the n number are arranged in order of size and there are an odd number of values. When there are an even number of values, the median is the mean of the two middle numbers.

The **mode** of the n numbers $x_1, x_2, \ldots, x_n$ is the number that occurs the most often. If no number occurs more often than any other number, there is no mode. If two numbers both occur the most often, then there are two modes.

Example

Find the mean, median and mode for the given sets of numbers

(a) 6, 3, 0, 9, 1, 9, 6, 0, 6, 6, 1, 9, 0, 3, 6, 1, 1, 7, 8, 7, 7, 4, 2, 9, 9

Mode is 7 and 9 (both occur 5 times)

$\bar{x} = 4.8 = \dfrac{120}{25}$ $\quad 120 = \Sigma x$

$25 = n$

(b) 6, 12, 3, 14, 9, 99

3 6 9 | 12 14 99

$M = \dfrac{9+12}{2} = 10.5$

no mode

mean $= \dfrac{143}{6}$ (≈ 23.8)

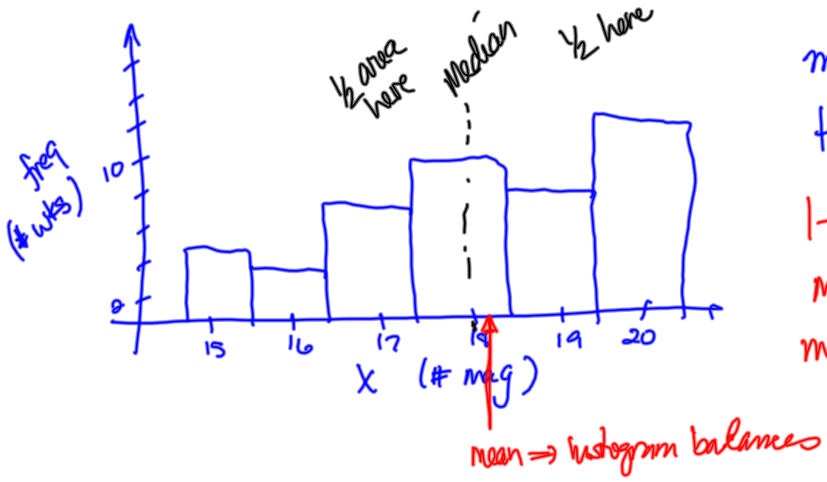
19, 15, 20, 20, 16, . . . } for 52 weeks of the year

Example: We are given the following data for the number of a certain magazine sold each week at a newsstand during the past year.

FREQ	# of weeks	5	4	8	11	9	15	L2
X	# of magazines	15	16	17	18	19	20	L1

(a) Represent this data in a frequency histogram.

(b) Find the mean, median, and mode for this data.



mode is 20 (highest freq)

tallest rectangle is the mode

1-var stats L1, L2

mean = 18.1538

median = 18

3, 7,

Example: A sample of grapefruits is selected from a large shipment and the number of seeds in each grapefruit is counted. The following results were found

FREQ	# grapefruits	6	7	8	9	10	L2
X	# seeds	7	6	5	4	3	L1

Find the mean, median, and mode for this data.

$\bar{x} = 4.75$, med = 5, mode = 3