WEEK 12 REVIEW (8.3 and 8.4)

8.3 Variance and Standard Deviation

3 3 3 3 3	2 2 3 4 4	0 0 5 5 5
$\mu = 3$	$\mu = 3$	$\mu = 3$

X	$P(X)$	$X-\mu$	$(X-\mu)^2$	$P(X)(X-\mu)^2$
2	1/5	−1	1	1/5
2	1/5	−1	1	1/5
3	1/5	0	0	0
4	1/5	1	1	1/5
4	1/5	1	1	1/5
TOTAL		0	4	4/5

VARIANCE $= \sum P(x)(x-\mu)^2 = 4/5$

STANDARD DEVIATION, $\sigma = \sqrt{var} = \sqrt{4/5}$

Example: Find the variance and standard deviation for the given sets of numbers: 6, 12, 3, 14, 9, 99 $\Rightarrow$ L1 ; 1-var Stats L1

$\sigma \approx 33.8103$ $var = \sigma^2 \approx 1143.13889 = \frac{41153}{36}$

Example: We are given the following data for the number of a certain magazine sold each week at a newsstand during the past year. What is the standard deviation in the number of magazines sold each week?

FREQ → L2

# of weeks	5	4	8	11	9	15
# of magazines	15	16	17	18	19	20

X → L1

1 Var Stats L1, L2 $\sigma \approx 1.6218$

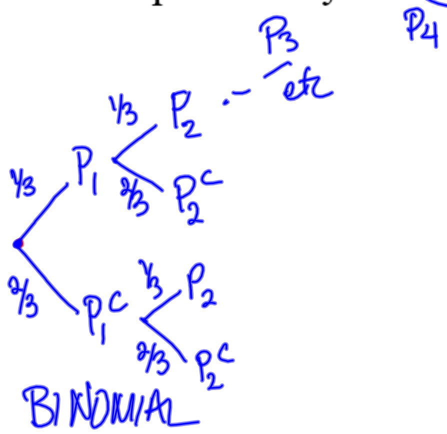
18, 19, 15, 17, 17, 19

8.4 The Binomial Distribution

In a Bernoulli trial we have the following:

- The same experiment repeated ~~several~~ a fixed # of time times.
- The only possible outcomes of these experiments are success or failure.
- The repeated trials are independent so the probability of success remains the same for each trial.

Example: In a certain neighborhood, 1/3 of the houses have swimming pools. If a random sample of 4 houses is chosen, what is the probability that exactly one of them has a pool?



$$P(P_1 \cap P_2^c \cap P_3^c \cap P_4^c) + P(P_1^c \cap P_2 \cap P_3^c \cap P_4^c) + \quad + \quad =$$

$$(1/3)(2/3)(2/3)(2/3) + (2/3)(1/3)(2/3)(2/3) + \_\_ + \_\_$$

$$4(1/3)^1(2/3)^3 = C(4,1)(1/3)^1(2/3)^{4-1}$$

$$= 0.3951$$

$$P(X=x) = C(n,x)(p)^x(1-p)^{n-x}$$

BINOMIAL PROBABILITY: If p is the probability of success in a single trial of a binomial (Bernoulli) experiment, the probability of x successes and n-x failures in n independent repeated trials of the same experiment is

Example: In a different neighborhood, 35% of the houses have a swimming pool and a random sample of 20 houses is chosen

(a) What is the probability that exactly 10 of the houses have pools?

DEFINE SUCCESS: HAVE A POOL

n = number of trials = 20

p = probability of success in a single trial = .35

x = number of successes = 10

binompdf(n, p, x) on the calculator: P(x = 10) = 0.0686

(b) What is the probability of at most 12 houses have pools?
\# of successes = x = 0, 1, 2, ... 12

$P(X \le 12) = P(x=0) + P(x=1) + P(x=2) + \dots P(x=12)$

binomcdf(n, p, x) is the sum of the probabilities from 0 to x.
binomcdf(20, .35, 12) = .9940

(c) What is the probability that more than 8 houses having pools?
* X = 9, 10, 11, ... 20 *

$1 = \underbrace{P(x=0) + P(x=1) + \dots + P(x=8)}_{\text{binomcdf}(20,\,.35,\,8)} + \underbrace{\boxed{P(x=9) + \dots \; P(x=20)}}_{\text{want this}}$

$P(9 \le x \le 20) = 1 - \text{binomcdf}(20, .35, 8) = 0.2376$

(d) What is the probability that between 10 and 20 houses have pools?

NON-INCLUSIVE

* $X = 11, 12, \ldots 19$ *

binomcdf(20, .35, 19) − binomcdf(20, .35, 10) = 0.0532

(e) What is the probability that 4 of the first 8 houses have pools and 5 of the last 12 houses have pools?

$\underbrace{\text{binompdf}(8, .35, 4)}_{\text{4 of first 8}} \underset{\text{and}}{\bullet} \underbrace{\text{binompdf}(12, .35, 5)}_{\text{5 of 12 have pools}} = .1875 \bullet .2039$

$= 0.03823$

$L_1 \rightarrow 0, 1, 2, \ldots 20$, $L_2 \rightarrow$ binomial prob $\Rightarrow$ 1varstats

(f) What is the expected number of houses that have pools? What is the standard deviation in the number of houses that have pools?

$\mu = 7$, $\sigma \approx 2.1331\ldots$

If X is a binomial random variable associated with a binomial experiment consisting of N trials with probability of success p in a single trial, then the mean (expected value) and standard deviation associated with the experiment are:

$\mu = Np$ and $\sigma = \sqrt{Np(1-p)}$

$\mu = 20 * .35 = 7$ $\sigma = \sqrt{20(.35)(1-.35)} = \sqrt{4.55} \approx 2.1331$