

**Probability Theory, Math 411 (section 503), Fall 2017 -
Homework 1**

From the textbook solve the problems 2,5,6 at the end of the Chapter 1 (pages 53-54).

And also the problems below:

Problem 1. What is the smallest (contained in all other) algebra which contains an event $A \subset \Omega$?

Problem 2. Construct a sample space Ω corresponding to tossing a die. Give example of two events that are disjoint. Give example of three events that are mutually disjoint, and whose union is Ω .

Problem 3. Given two events A and B with $P(A) = 0.4$ and $P(B) = 0.7$, what are the maximum and minimum possible values for $P(A \cap B)$?

Problem 4. Show that for any sets A and B

$$\mathbf{P}(A \cap B) \leq \mathbf{P}(A) \leq \mathbf{P}(A \cup B).$$

Problem 5. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely. (a) Find the probability that doubles are rolled. (b) Given that the roll results in a sum of 4 or less, find the conditional probability that doubles are rolled. (c) Find the probability that at least one die roll is a 6.

Problem 6. Two dice are rolled. What is the probability that (a) the two numbers will differ by 1 or less and (b) the maximum of the two numbers will be five or larger?

Problem 7. We have a very weird dice, for which we know that the probability that the number is strictly less than 5 is 0.6, the probability that we get 4 is equal to 0.2 and the probability that the number is strictly bigger than 2 is 0.9. Show that this information is not sufficient to determine the probability law, that is there is more than one probability law for this experiment which has the above properties.

Problem 8. You want to buy a car on a certain website. During the purchase they first ask you for the model (you can choose between 3 models: A, B and C), then they ask you for the engine type (you have two options: normal and hybrid), then they ask you for the transmission type (you have

two options: manual and automatic), and finally they ask you to choose one of 5 different colors. In how many ways can you configure your car, if you know that the only restriction is that hybrid must have an automatic transmission.

Problem 9. Let A, B be two events in a probability space. Show that the probability that either A or B has occurred, but not both, is $P(A) + P(B) - 2P(A \cap B)$.