

Mathematical Probability, Math 411 - Homework 10

From the textbook solve the problems 17,18,19, 22, 23 and 24 from the Chapter 4.

Solve the problems 21, 22, 24, 30 from the Chapter 4 additional exercises at

<http://www.athenasc.com/prob-supp.html>

And also the problems below:

Problem 1. Show that for random variables X , Y and Z we have

$$\mathbf{E}[\mathbf{E}[\mathbf{E}[X|Y]|Z]] = \mathbf{E}[X].$$

Apply this formula to the following problem: Roll a fair 6-sided die and observe the number Z that came up. Then toss a fair coin Z times and observe the number of heads Y . Then let X be a number uniformly chosen in the interval $[0, Y]$. Find $\mathbf{E}[X]$.

Problem 2. A machine prints out the number 1, 2, or 3 with equal probabilities. A machine starts printing numbers independently one after another and stops the first time it prints 1. Let X denote the number of 2s it printed. Compute $\mathbf{E}[X]$ and $\text{var}(X)$.