

Mathematical Probability, Math 411 - Homework 6

From the textbook solve the problems 11,12, 13, 14, 15,16,18 from the end of chapter 3. And also the problems below:

Problem 1. Which of the following functions can be a probability density function for some continuous random variable. Explain your reasoning.

1)

$$f(t) = \begin{cases} e^t + 4t - e, & \text{for } 0 \leq t \leq 1 \\ 0, & \text{otherwise.} \end{cases}$$

2)

$$f(t) = \begin{cases} e^{-t}, & \text{for } t \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

3)

$$f(t) = e^{\sin(t)}, \text{ for all } t \in \mathbb{R}.$$

4)

$$f(t) = \begin{cases} 0, & \text{for } t < 0 \\ \frac{1}{2}, & \text{for } 0 \leq t \leq 1 \\ 0, & \text{for } 1 < t < 2 \\ \frac{1}{2}, & \text{for } 2 \leq t \leq 3 \\ 0, & \text{for } 3 < t. \end{cases}$$

Problem 2. Continuous random variable X has probability density function $f_X(t)$ such that

$$f_X(t) = \begin{cases} C \sin(\pi t/n), & \text{for } 0 \leq t \leq n \\ 0, & \text{otherwise.} \end{cases}$$

Find the value of the constant C . Then find $\mathbf{E}(X)$ and $\text{var}(X)$.

Problem 3. Let $p > 0$ be a real number. Consider function f which has values $f(t) = Ct^{-p}$ for $t \geq 1$, and $f(t) = 0$ for $t < 1$. For which values of p we can select constant C so that this function f is a probability density function for some continuous random variable? For which values of p will this random variable have finite expectation? For which values of p will this random variable have finite variance?