Note Title 8/3/2015

Completions

Notice that it's clear that every subset A of a metric space (M, d) has a closure. I.e., if A is not closed, it's because there exists some sequence (x_n) that converges to $x \in M \setminus A$. But we create $\bar{A}$ simply by adding all such points to A.

We can ask a similar question about completeness: given a metric space (M,d), can we complete it (create its completion)? The answer is, always, yes.

Definition

A metric space $(\hat{M}, \hat{d})$ is called a completion for (M,d) if:

(i) $(\hat{M}, \hat{d})$ is complete

(ii) (M, d) is isometric to a dense subset of $(\hat{M}, \hat{d})$.

Condition (ii) is a technical way of saying that we can view M as a subset of $\hat{M}$ (so, as with closing a set, we've added some points to get $\hat{M}$), and $\hat{d}$

can be regarded as the same as d for all elements of M.