Note Title 8/6/2015

Compactness

Definition

We say a metric space (M,d) is **compact** if it is both complete and totally bounded.

Examples

1. A subset K of $\mathbb{R}^n$ is compact if and only if it is closed and bounded. This is

because: (1) by Theorem 7.9 K is complete iff it is closed; and (2) by a generalization of Problem 7.2 K is is bounded iff it is totally bounded.

2. The set $\{e_n : n \geq 1\}$ of unit vectors in ℓ_∞ is closed and bounded, but we've seen that it is not totally bounded (p. 90 in Carothers). So the correspondence betwee compact

and closed and bounded is <u>not</u> true for all metric spaces.

3. A subset of a discrete is compact iff it is finite. This is because a discrete space with an infinite number of points cannot be totally bounded (see Carothers, p. 90), but any finite number of points is totally bounded. Also, every discrete space is

complete (Carothers, p. 92).