Overview and Least Upper Bound Axiom

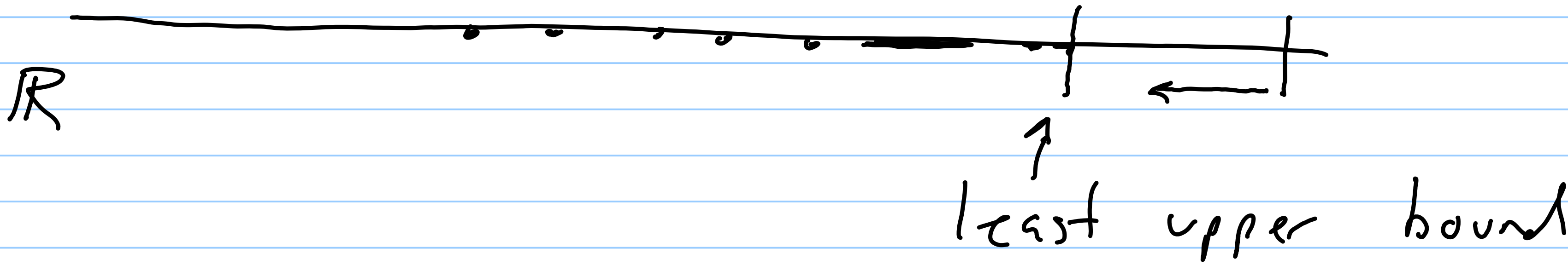
Note Title 6/7/2015

First, recall that a subset A of the real numbers $\mathbb{R}$ (i.e., $A \subset \mathbb{R}$) is said to be bounded above if there exists some $x \in \mathbb{R}$ so that $a \leq x$ for all $a \in A$.

In this case, x is referred to as an upper bound for A.

Least Upper Bound Axiom

Any nonempty set of real numbers with an upper bound has a least upper bound.



Example

For $(0,1)$ the l.u.b. is 1, and similarly for $[0,1]$. We could also have a sequence of points such as $\{1-\frac{1}{n}\}_{n=1}^{\infty}$. In this case the l.u.b. is 1.

For sets like this, we express the least upper bound as the supremum, and we write

$$\sup (0,1) = 1$$

$$\sup [0,1] = 1$$

$$\sup \left\{ 1 - \frac{1}{n} \right\}_{n=1}^{\infty} = 1.$$