Note Title 6/10/2015

Continuity

Definition

If f is defined in a neighborhood of a (i.e., an open interval containing a) we say f is continuous at a if

$$\lim_{x \to a} f(x) = f(a).$$

We denote by

$$\lim_{x \to a^-} f(x)$$

the limit as x approaches a from values smaller than a. Likewise we denote by

$$\lim_{x \to a^+} f(x)$$

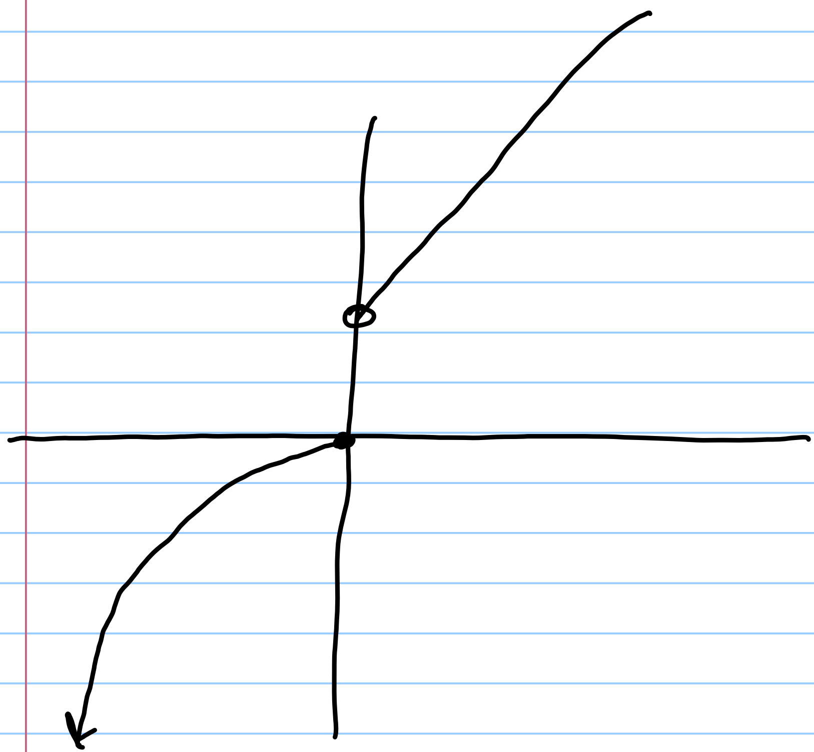
the limit as x approaches a from values larger than a.

Carothers often denotes the first of these $f(a^-)$ and the second $f(a^+)$.

If $f(a^-)$ and $f(a^+)$ both exist, but have different limits, we say f has a jump discontinuity at a.

Example

Consider $f(x) = \begin{cases} -x^2 & x \le 0 \\ x+1 & x > 0 \end{cases}$



$$\lim_{x \to 0^-} f(x) = 0$$

$$\lim_{x \to 0^+} f(x) = 1 \ .$$

Theorem 1.18

Let f be a real-valued function defined in some neighborhood of $a \in \mathbb{R}$. Then the following are equivalent:

(i) f is continuous at a

(ii) $\lim_{n\to\infty} x_n = a \Rightarrow \lim_{n\to\infty} f(x_n) = f(a)$

(iii) $\lim_{n\to\infty} x_n = a \Rightarrow \lim_{n\to\infty} f(x_n)$ converges to something

(iv) $\lim_{x \to a^-} f(x) = f(a)$ and $\lim_{x \to a^+} f(x) = f(a)$.