Note Title 6/10/2015

Set Equivalence

Consider the sets

$$A = \{1, 2, 3\}$$

$$B = \{1, 4, 9\}.$$

We'll say these sets are equivalent, because they have the same number of elements.

Definition (Set Equivalence)

We say two sets A and B are equivalent if there is a 1-1 correspondence between them. Many authors refer to these as equipotent sets.

For example, if

$A = \{1, 2, 3, \ldots\}$, $B = \{1, 4, 9, \ldots\}$

We say A and B are equivalent.

Perhaps more curious, let

$$A = (0,1)$$

$$B = (0,2)$$

The function $f(x) = 2x$ gives a 1-1 correspondence between these sets, so we say they are equivalent.

We refer to this measure of the number of elements of a set as the set's <u>cardinality</u>, so we would say A and B have the same cardinality.

An infinite set A is said to be <u>countable</u> if we can enumerate all of its elements. For example, the following sets are all countable and equivalent:

$$\{1, 2, 3, \ldots\}$$

$$\{1, \tfrac{3}{2}, 2, \tfrac{5}{2}, 3, \ldots\}$$

1 2 3 4 ...

$$\{1, \tfrac{5}{4}, \tfrac{3}{2}, \tfrac{7}{4}, 2, \ldots\}$$