

# Power Sets

Note Title

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## Definition

Given any set  $A$ , we denote by  $P(A)$  the power set of  $A$ , which is the set of all subsets of  $A$ , including  $A$  and  $\emptyset$ .

For example, suppose  $A = \{1, 2, 3\}$ .

The subsets of  $A$  are:  
 $\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}$ .

In general, if  $A$  has  $n$  elements, its power set has  $2^n$  elements.

This is easy to see, because the number is:

$$\binom{n}{0} + \binom{n}{1} + \binom{n}{2} + \dots + \binom{n}{n}$$

empty set      singletons      sets with 2 elements      etc.       $\uparrow$  set itself

$$= \sum_{j=0}^n \binom{n}{j} = (1+1)^n \text{ by the binomial theorem.}$$
$$= 2^n$$

Here,

$$\binom{n}{j} = \frac{n!}{j!(n-j)!}$$

The Binomial Theorem asserts:

$$(x + y)^n = \sum_{j=0}^n \binom{n}{j} x^j y^{n-j}$$

How large is  $P(N)$ ? We'll answer this in the next few lectures.