## MATH 609-600

## Homework \#4

## Polynomial, Birkhoff, and Spline Interpolation

The homework should be presented at the beginning of the class. Penalty for delaying the homework is 5 pts per day.
(1) (20 pts) (Kincaid and Cheney, Section 6.2, problem 23) The polynomial $p(x)=$ $2-(x+1)+x(x+1)-2 x(x+1)(x-1)$ interpolates the first four points in the table $(-1,2),(0,1),(1,2),(2,-7),(3,10)$. By adding one additional term to $p$, find polynomial that interpolates the whole table.
(2) (20 pts) Find the algebraic polynomial of the lowest degree (using divided differences) that interpolates the data

$$
x=0, f(0)=0, f^{\prime}(0)=0, f^{\prime \prime}(0)=1, x=1, f(1)=1, f^{\prime}(1)=0, f^{\prime \prime}(1)=1
$$

and prove that the polynomial is unique.
(3) (20 pts) Find the periodic cubic spline that interpolates the data: $(0,0),\left(\frac{1}{2}, 1\right),(1,0)$.
(4) (20 pts) (Kincaid and Cheney, Section 7.1, problem 16)
(5) (20 pts) (Kincaid and Cheney, Section 7.2, problem 23)

