MATH 609-602
Homework #1
Matrix algebra and direct methods for linear systems

Solve any set of problems for 100 points.

1. (20 pts) (problem 11, p. 157 of your textbook) Prove that the inveres of a nonsingular upper triangular matrix is also upper triangular.

2. (20 pts) (problem 12, p. 157 of your text) Let A be an $n \times n$ invertible matrix, and let u and v be two vectors in R^n. Find the necessary and sufficient conditions on u and v in order that the matrix

$$\begin{bmatrix} A & u \\ v^T & 0 \end{bmatrix}$$

to be invertible, and give a formula for the inverese when it exists.

3. (20 pts) (problem 7b, p. 169 of your text) Write the **column version** of the Doolittle algorithm, which computes the k-th column of L and the k-th column of U at the k-th step. (Consequently, the order of computing is $u_{1k}, u_{2,k}, ..., u_{kk}, l_{k+1,k}, ..., l_{nk}$ at the k-th step.) Count the number of arithmetic operations (additions/subtractions and multiplications/divisions separately).

4. (20 pts) (problem 26, p. 170 of your text) Prove: A is positive definite and B is nonsingular if and only if BAB^T is positive definite.

5. (20 pts) (problem 27, p. 170 of your text) If A is positive definite, does it follow that A^{-1} is also positive definite ?

6. (20 pts) (problem 16, p. 158 of your text) For what values of a is this matrix positive definite ?

$$A = \begin{bmatrix} 1 & a & a \\ a & 1 & a \\ a & a & 1 \end{bmatrix}$$

7. (20 pts) Discuss the positive definiteness of the following three matrices:

$$A = \begin{bmatrix} 3 & 2 & 0 & 1 \\ 2 & 5 & 2 & 1 \\ 0 & 2 & -1 & 0 \\ 1 & 1 & 0 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 2 & 0 & 0 \\ 2 & 4 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 2 \end{bmatrix} \tag{1}$$