**MATH 609 Numerical Analysis**
**Programming assignment #2**
**Basic iterative methods for linear systems**

## 1. Problem Formulation

Write a program for solving the system $Ax = b$ by 1) Jacobi, 2) Gauss-Seidel, 3) SOR (with some choice of the parameter $\omega$, you can experiment with the choice), and 4) SSOR. Use the standard for SOR splitting of $A$, namely, $A = D - L - U$, where $D$ is the diagonal of $A$, $L$ is strictly lower triangular and $U$ strictly upper triangular matrices. Do not store the matrix $A$.

## 2. Specifications

(1) For stopping the iterations use any of the conditions:

$$\|x^{(m+1)} - x^{(m)}\|_2/\|x^{(0)}\|_2 < TOL, \qquad \|r^{(m)}\|_2/\|r^{(0)}\|_2/ < TOL,$$

where $r^{(m)} = b - Ax^{(m)}$ is the residual of the $m$-th iterate, or any other criterion you consider appropriate. Set $TOL = 10^{-6}$ or $TOL = 10^{-12}$ if using double precision.

(2) Solve the following linear systems:

Example 1:
$x_0 = 0,\ k_{i-1}(x_i - x_{i-1}) + k_i(x_i - x_{i+1}) + c_i x_i = b_i,\ i = 1, ..., n,\ x_{n+1} = 0$, which after the elimination of $x_0$ and $x_{n+1}$ is written in the form $Ax = b$ with $x \in R^n$. Solve for the following systems for $n = 20,\ 40$:

(1) $k_i = 1$, $i = 0, \ldots, n$, $c_i = 0$ and $b_i = 1/(n+1)^2$, $i = 1, \ldots, n$;

(2) $k_i = 1$, $i = 0, \ldots, n/2$ and $k_i = 10$, $i = n/2+1, \ldots, n$ and $c_i = 0.1$ and $b_i = 1/(n+1)^2$, $i = 1, \ldots, n$;

Remark. The solution $x_i$ of (1) is an approximation of $u(t_i)$, $t_i = i/(n+1)$ with $u(t)$ the solution of a boundary value problem $-u'' + cu = b$ on the interval $(0, 1)$ with boundary conditions $u(0) = u(1) = 0$. Then plot $x_i$.

Example 2: The unknowns are given as a two dimensional array $x_{ij}, i, j = 0, \ldots, n+1$ that satisfy the system

(2) $$(4 + h^2)x_{i,j} - x_{i-1,j} - x_{i+1,j} - x_{i,j-1} - x_{i,j+1} = h^2, x_{0,j} = x_{n+1,j} = x_{i,0} = x_{i,n+1} = 0.$$

Here $h = 1/(n+1)$ so that the system represents a finite difference approximation of the boundary value problem $-\Delta u + u = 1$ in $\Omega = (0, 1) \times (0, 1)$ and $u = 0$ on the boundary of $\Omega$. Take $n = 8,\ 16,\ 32$.

(3) In all problems take a r.h.s. $b = h^2(1, 1, \ldots, 1)^t$ and $x^0 = (0, 0, \ldots, 0)^t$ or $x^0 = b$.

(4) The report should be in the specified latex-format (or MS doc format) and must contain tables with the number of iterations for all cases (plots are welcome as well). Compare the number of iteration for these iteration methods.

*Remark.* If you are getting to big differences in the iteration count then there is something wrong either with your implementation or with the estimates of the maximum and minimum eigenvalues of $A$ you have used.

The penalty for delaying the programming assignment is 5 pts per day (out of 100).