MATH 609 Numerical Analysis
Programming assignment #3
Steepest Descent and Conjugate Gradient iterative methods

1. Problem Formulation

Write a program for solving the system $Ax = b$ (generated by Examples 1 and 2 of the Programming assignment # 2) by Steepest Descent (SD) and Conjugate Gradient (CG) iterative method. Make comparison of the results in terms of iteration count.

2. Specifications

(1) Solve the following linear systems:

Example 1:

$x_0 = 0,\ k_{i-1}(x_i - x_{i-1}) + k_i(x_i - x_{i+1}) + c_i x_i = b_i,\ i = 1, ..., n,\ x_{n+1} = 1$, which after the elimination of x_0 and x_{n+1} is written in the form $Ax = b$ with $x \in R^n$. Solve for $n = 25,\ 50,\ 100$. The rest of the data you can borrow from the previous Programming Assignment.

Example 2: The unknowns are given as a two dimensional array $x_{ij}, i, j = 0, \ldots, n+1$ that satisfy the system

$$(4 + h^2)x_{i,j} - x_{i-1,j} - x_{i+1,j} - x_{i,j-1} - x_{i,j+1} = h^2, x_{0,j} = x_{n+1,j} = x_{i,0} = x_{i,n+1} = 0.$$

Here $h = 1/(n+1)$ so that the system represents a finite difference approximation of the boundary value problem $-\Delta u + u = 1$ in $\Omega = (0,1) \times (0,1)$ and $u = 0$ on the boundary of Ω. Take $n = 8,\ 16,\ 32$.

(2) In all problems take a r.h.s. $b = h^2(1, 1, \ldots, 1)^t$ and $x^0 = (0, 0, \ldots, 0)^t$ or $x^0 = b$.

(3) Let $r^m = b - Ax^m$ be the residual of the m-iterate. Stop the iteration process when $\|r^m\|/\|r^0\| \le TOL$, where $\|x\|^2 = \sum |x_i|^2$ for any $x \in R$ and set $TOL = 10^{-6}$. You may experiment with $TOL = 10^{-12}$ if using double precision.

(4) The report should be in the specified format and should contain tables (report only for $n = 50$) in both examples. In order to have a chance to compare the iteration count with the results of Jacobi and SOR, simply add two new columns to the previous tables.

(5) Since the solutions x_i and $x_{i,j}$ of these systems represent approximations of the solutions $u(x_i)$ and $u(ih, jh)$ of some boundary value problems, it makes sense to plot them as functions in one and two variables, respectively. Present graphs only for the finest grids.

3. Penalties

The penalty for delaying the programming assignment is 5 pts per day (out of 100).