MATH 141 EXAM 1, VERSION A SPRING 2010

"On my honor, as an Aggie, my signature shows that
I have neither given, nor received, unauthorized aid on this test."

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- Check to see that you have 5 pages front/back.
- This exam consists of 7 multiple choice problems, 7 problems in the work out section and the TRUE/FALSE part. Clearly indicate your answers to the multiple choice questions on your exam.
- **You must show all appropriate work to receive full credit.**
- Be sure to read the instructions to each problem *carefully.*
- Use a pencil and be neat. On the workout problems, if I can't read your answers, then I can't give you credit.
- There are 100 points possible. Point values for each problem are as indicated.
- **You must clear your calculator: MEM (2nd +), Reset (7), cursor right to ALL, All Memory (1), Reset (2).**
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

Good Luck!

1-7	8	9	10	11	12	13	14	15	16	GRADE
28	10	5	12	5	9	9	4	10	8	100

Part I - Multiple choice Please clearly circle your answers. Partial credit will not be given for these questions.

1. [4pts] In 2005 a company installed a new machine in one of its factories at a cost of \$250,000. The machine is depreciated linearly over 10 yr with a scrap value of \$10,000. Find the machine's book value in 2010.

   (a) \$178,000
   (b) \$154,000
   c (c) \$130,000
   (d) \$106,000
   (e) none of these

2005	$t=0$	\$250,000
2015	$t=10$	\$10,000
2010	$t=5$	?

$m = \frac{250,000-10,000}{0-10} = -24,000$

$v(t) = -24,000t + 250,000 \Rightarrow V(5) = -24,000\cdot 5 + 250,000 = 130,000$

2. [4pts] An analyst has found that the demand equation for the sale of strawberries each day in a particular city is $0.2x + p - 4 = 0$ and the supply equation is $p = 0.07x + 0.76$, where p is the price of the strawberries in dollars per pound and x is the number of strawberries in thousand of pounds. What is the equilibrium quantity and price?

   (a) 12 pounds, \$1.60
   (b) 12,000 pounds, \$840.76
   c (c) 12,000 pounds, \$1.60
   (d) 1,600 pounds, \$12
   (e) none of these

demand: $p = -0.2x + 4$
supply $p = 0.07x + 0.76$
Intersection point (using calculator)
$(x, y) = (12, 1.6) \rightarrow$ 12,000 & \$1.6

3. [4pts] Let $A = \begin{bmatrix} -4 & -6 & 2 \\ 5 & 3 & -1 \\ -2 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1/6 & 1/3 & 0 \\ -1/14 & 0 & 1/7 \\ 13/21 & 2/3 & 3/7 \end{bmatrix}$.

   One of the statements below is NOT true. Find it.

   (a) $B = A^{-1}$
   (b) $A = B^{-1}$
   (c) $A(B + A) = AB + A^2$
   (d) $AB = I$
   e (e) $AB \neq BA$

4. [4pts] The matrix below is already in RREF. What would the solution be for this system of linear equations?

$$\begin{array}{cccc|c} x & y & z & w & \\ \hline 1 & 0 & 0 & 0 & 6 \\ 0 & 1 & 3 & 5 & 6 \end{array}$$

$z = t$
$w = s$
$x = 6$
$y = 6 - 3z - 5w = 6 - 3t - 5s$

   (a) $(x, y, z, w) = (6, 6, 0, 0)$
   (b) $(x, y, z, w) = (6, 6 - 3t, t, 0)$, where t is a real number.
   c ~~(c) $(x, y, z, w) = (6, 6 - 3t - 5s, t, s)$, where t and s are real numbers.~~
   (d) no solution.

5. [4pts] Matrix F shows the number of each type of flower that was sold at a flower shop on three different days (Monday (M), Tuesday (T) and Wednesday (W)). Given that roses cost \$5 each, sunflowers cost \$2 each, gardenias cost \$3 each. Find matrix P such that the product of the two matrices shows revenue from flowers that were sold on each day.

$$F = \begin{array}{c c} & \begin{array}{ccc} M & T & W \end{array} \\ \begin{array}{c} roses \\ sunflowers \\ gardenians \end{array} & \begin{bmatrix} 30 & 20 & 40 \\ 50 & 25 & 42 \\ 44 & 12 & 18 \end{bmatrix} \end{array}$$

(a) $P = \begin{bmatrix} 5 & 2 & 3 \end{bmatrix}$ with product FP

(b) $P = \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}$ with product FP

(c) $P = \begin{bmatrix} 5 \\ 2 \\ 3 \end{bmatrix}$ with product PF

d (d) $P = \begin{bmatrix} 5 & 2 & 3 \end{bmatrix}$ with product PF

(e) none of these

F = [flowers × day]
P = [price × flower]
We need [price × day]
⇒ PF

6. [4pts] Let $A = \begin{bmatrix} 3 & -2 \\ 4 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 4 & -5 \\ p & -6 \end{bmatrix}$, $C = \begin{bmatrix} k & -3 \\ 1 & -56 \end{bmatrix}$.

If $AB = C$, find the value of k .

(a) 7

(b) 11.5

c (c) 17

(d) 27

(e) none of these

$AB = \begin{bmatrix} 12-2p & -15+12 \\ 16+6p & -20-36 \end{bmatrix} = \begin{bmatrix} k & -3 \\ 1 & -56 \end{bmatrix}$

$12-2p = k$

$16+6p = 1 \Rightarrow 6p = -15 \Rightarrow 2p = -5$

$k = 12-2p = 12-(-5) = 17$

7. [4pts] Is the augmented matrix in reduced row echelon form?

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 1 & b \\ 0 & 0 & 0 & 0 \end{array}\right]$$

(a) No, because the leading ones are not properly arranged.

(b) No, because the second row creates a contradiction.

C (c) Yes, the matrix is in row-reduced form.

(d) None of these.

Part II - Work Out Problems You must show all appropriate work for all problems in this section to receive full credit. If you do the work in your head, you must turn in your head to get full credit. Include any intermediate steps and programs/functions you use on your calculator.

8. [10pts] Let $A = \begin{bmatrix} 3 & a & -1 \\ 6 & e & 4 \end{bmatrix}$, $B = \begin{bmatrix} b & -4 \\ 2 & -1 \\ -3 & c \end{bmatrix}$, $C = \begin{bmatrix} -6 & 1 & d \\ -2 & -7 & 5 \end{bmatrix}$.

Consider the equation $A^{\top} + 2B = C^{\top}$. Find values of a, b, c, d and e.

SHOW WORK HERE

$$\begin{bmatrix} 3 & 6 \\ a & e \\ -1 & 4 \end{bmatrix} + 2\begin{bmatrix} b & -4 \\ 2 & -1 \\ -3 & c \end{bmatrix} = \begin{bmatrix} -6 & -2 \\ 1 & -7 \\ d & 5 \end{bmatrix}$$

$$\begin{bmatrix} 3+2b & 6-8 \\ a+4 & e-2 \\ -1-6 & 4+2c \end{bmatrix} = \begin{bmatrix} -6 & -2 \\ 1 & -7 \\ d & 5 \end{bmatrix}$$

$3+2b=-6 \Rightarrow 2b=-9 \Rightarrow b=-9/2$

$a+4=1 \Rightarrow a=1-4=-3$

$-7=d$

$e-2=-7 \Rightarrow e=-7+2=-5$

$4+2c=5 \Rightarrow 2c=5-1 \Rightarrow c=\frac{1}{2}$

WRITE ANSWERS HERE

$a = \underline{-3}$, $b = \underline{-9/2}$, $c = \underline{\frac{1}{2}}$, $d = \underline{-7}$, $e = \underline{-5}$

9. [5pts] Ariel has three times as many nickels as quarters and three more dimes than nickels. If the total face value of these coins is \$2.40, how many coins does Ariel have? **Set up the system of equations, but DO NOT SOLVE.** Be sure to clearly define the variables.

n # of nickels $\quad n = 3q$

d # of dimes $\quad d = n + 3$

q # of quarters $\quad 0.05n + 0.1d + 0.25q = 2.4$

10. Sean is selling fruit for a school fundraiser. Customers can buy small boxes of oranges, medium boxes of oranges and large boxes of oranges. The prices for small, medium and large box are \$9, \$12 and \$15 respectively. Sean sold the boxes of oranges for a total of \$105. If Sean sold 9 boxes in total and the number of small boxes was equal to the number of large boxes plus one, how many of each size boxes did he sell?

(a) [2pts] Define the variables that are used in setting up the system of equations.

x # of small boxes
y # of medium boxes
z. # of large boxes

(b) [3pts] Set up the system of equations that represents this problem.

$$x+y+z=9$$
$$9x+12y+15z=105$$
$$x=z+1$$

(c) [7pts] Solve for the solution. *If the solution is unique list it. If NO SOLUTION, say so. If more than one solution, write it in parametric form, then tell what restrictions should be placed on the parameter(s); also give one specific solution.*

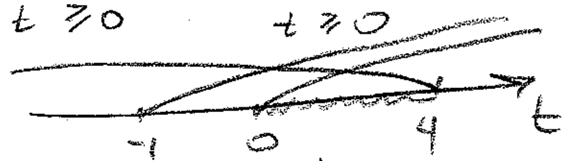
Augmented matrix

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 9 \\ 9 & 12 & 15 & 105 \\ 1 & 0 & -1 & 1 \end{array}\right] \xrightarrow{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 1 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

x, y, z must be nonnegative integers

$$x-z=1$$
$$y+2z=8 \rightarrow \boxed{\begin{array}{l} x=t+1 \\ y=8-2t \\ z=t \end{array}}$$
$$z=t$$

$t+1\geq 0 \rightarrow t\geq -1$
$8-2t\geq 0 \quad t\leq 4$
$t\geq 0 \quad t\geq 0$



$0\leq t\leq 4$

$\boxed{t=0,1,2,3,4}$

specific solution

$t=0 \Rightarrow (x,y,z)=\boxed{(1,8,0)}$

1 small box, 8 medium, 0 large

11. Use this system of equations to answer the next two questions:

$$\begin{aligned} x + 2y &= 7 - 3z \\ -x + 2z &= 5 \\ 5x + 3y &= 3 + z \end{aligned} \quad \rightarrow \quad \begin{aligned} x + 2y + 3z &= 7 \\ -x + \quad 2z &= 5 \\ 5x + 3y - z &= 3 \end{aligned}$$

(a) [2pts] What is the coefficients matrix?

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 2 \\ 5 & 3 & -1 \end{bmatrix}$$

(b) [3pts] Compute the inverse matrix and express it with exact values (fractions). If this is not possible, then explain why.

$$A^{-1} \begin{bmatrix} -2 & 11/3 & 4/3 \\ 3 & -16/3 & -5/3 \\ -1 & 7/3 & 2/3 \end{bmatrix}$$

12. [9pts] Daniel manages a heaters store which has a monthly rent \$900. Daniel needs to sell 45 heaters each month to break even and he sells the heaters for \$30. Use the given information to find the linear functions for total cost, revenue, and profit. Show all work, and record your final answers in the blanks provided.

$F = 900$ $\qquad R(x) = sx = 30x$

$P(45) = 0$ $\qquad C(x) = cx + F = cx + 900$

$s = 30$ $\qquad P(x) = R(x) - C(x) = 30x - cx - 900$

$P(45) = 0 \Rightarrow 30 \cdot 45 - c \cdot 45 - 900 = 0$

$30 - c - 20 = 0$

$c = 10$

$C(x) = 10x + 900$

$P(x) = 30x - 10x - 900$

$P(x) = 20x - 900$

Total cost function: $C(x) = 10x + 900$

Revenue function: $R(x) = 30x$

Profit function: $P(x) = 20x - 900$

13. A study of the environmental impact on black bass in large lakes was conducted. The number of fish (in thousands) within one-half mile of the source of pollution was estimated when various amounts of a particular pollutant were introduced into the water. The following data were collected:

Number of tons of pollutant, (x)	20	30	40	50	60	70
Thousands of fish, (y)	40	33	30	26	20	16

(a) [3pts] Find the least squares regression equation for the data, assuming it is linear. (*Round to* **four** *decimal places.*)

$y = -0.4657x + 48.4571$

(b) [3pts] Use the result of part (a) to estimate the black bass population if 54 tons of the pollutant were introduced. (*Round to the nearest integer.*)

$x = 54 \Rightarrow y = -0.4657 \cdot 54 + 48.4571 = 23.3093 \approx 23$

Answer: 23 thousands of fish

or 23309

(c) [3pts] If the black bass population is estimated at 28,000 fish, how many tons of pollutants can be assumed to have been put into the lake? (*Round to* **two** *decimal places.*)

$y = 28 \Rightarrow 28 = -0.4657x + 48.4571$

$$x = \frac{28 - 48.4571}{-0.4657} = 43.93$$

Answer: 43.93 tons of pollutant

14. [4pts] Fill in the missing entries by performing the indicated row operations to obtain the row-reduced matrices.

$$\left[\begin{array}{cc|c} 1 & 6 & -7 \\ 1 & 1 & 0 \end{array}\right] \xrightarrow{R_2 - R_1} \left[\begin{array}{cc|c} 1 & 6 & -7 \\ 0 & -5 & 7 \end{array}\right] \xrightarrow{-\frac{1}{5}R_2} \left[\begin{array}{cc|c} 1 & 6 & -7 \\ 0 & 1 & -7/5 \end{array}\right] \xrightarrow{R_1 - 6R_2} \left[\begin{array}{cc|c} 1 & 0 & \frac{7}{5} \\ 0 & 1 & -\frac{7}{5} \end{array}\right]$$

Part III True/False

15. [2pts each] If A is a 3×4 matrix, B is a 4×4 matrix and C is 4×3 matrix.
CLEARLY circle TRUE or FALSE for each statement. If the operation is not possible, .
If the operation is possible, **GIVE THE SIZE** *of the resulting matrix.*

TRUE FALSE It is possible to compute $4(A + C^T)$.

$A + C^T \to 4(A+C^T)$
$3\times 4 \quad 3\times 4 \quad \boxed{3 \times 4}$

TRUE FALSE It is possible to compute ABC.

$A \quad B \quad C \to \boxed{3\times 3}$
$3\times 4 \quad 4\times 4 \quad 4\times 3$

TRUE FALSE It is possible to compute CAB.

$C \quad A \quad B \to \boxed{4\times 4}$
$4\times 3 \quad 3\times 4 \quad 4\times 4$

TRUE FALSE It is possible to compute $B(A - 2C)$.

$A - 2C$ DNE because A and C are not of the same size
$3\times 4 \quad 4\times 3$

TRUE FALSE It is possible to compute $A^{-1} + C$.

A^{-1} DNE because A is not square

16. [1pt each] Given five lines:

$L_1: \ 2x + 3y - 5 = 0 \qquad L_3: \ y = 2010 \qquad L_5: \ y - 3x + 30 = 0$
$L_2: \ y = \frac{3}{2}x + 12 \qquad L_4: \ x = 2010$

CLEARLY circle TRUE or FALSE for each statement.

$L_1: \ y = -\frac{2}{3}x + \frac{5}{3}$

TRUE FALSE L_1 and L_2 are parallel.

$m_1 = -\frac{2}{3}$

TRUE FALSE L_1 and L_5 are parallel.

$m_2 = \frac{3}{2}$

TRUE FALSE L_3 and L_4 are perpendicular.

$m_3 = 0$
m_4 undef.

TRUE FALSE L_1 and L_2 are perpendicular.

$m_5 = 3$

TRUE FALSE L_5 has negative slope.

TRUE FALSE The points $(10, 0)$ and $(0, 30)$ are intercepts of L_5.

TRUE FALSE L_3 has undefined slope.

TRUE FALSE L_4 passes through the point $(2010, 2010)$ and parallel to the y-axis.