MATH 141 EXAM 2 VERSION A

Printed LAST NAME______________ FIRST NAME______________

"On my honor, as an Aggie, my signature shows that
I have neither given, nor received, unauthorized aid on this test."

SIGNATURE:__________________ UIN__________________ Seat Number______

- Check to see that you have 4 pages front/back.
- This exam consists of 8 multiple choice problems, 7 problems in the work out section and the TRUE/FALSE part. Clearly indicate your answers to the multiple choice questions on your exam.
- **You must show all appropriate work to receive full credit.**
- Be sure to read the instructions to each problem *carefully.*
- Use a pencil and be neat. On the workout problems, if I can't read your answers, then I can't give you credit.
- There are 100 points possible. Point values for each problem are as indicated.
- **You must clear your calculator: MEM (2nd +), Reset (7), cursor right to ALL, All Memory (1), Reset (2).**
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

Good Luck!

1-8	9	10	11	12	13	14	15	16	GRADE
40	6	8	8	6	10	6	8	8	100

Part I - Multiple choice Please clearly circle your answers. Partial credit will not be given for these questions.

1. [5pts] Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and let $A = \{x \in U | x$ is a multiple of 3$\}$ and $B = \{2, 5, 6\}$. Find $B \cap A^c$.

   (a) $\{1, 2, 4, 5, 6, 7, 8, 10\}$

   (b) $\{6\}$

   C (c) $\{2, 5\}$

   (d) $\{1, 2, 3, 4, 5, 7, 8, 9, 10\}$

   (e) none of these

   $A^c = U - \{3, 6, 9\} =$
   $= \{1, 2, 4, 5, 7, 8, 10\}$
   $B \cap A^c = \{2, 5\}$

2. [5pts] Given $P(E) = 0.37$, what is $P(E^c)$?

   a (a) 0.63

   (b) 0.315

   (c) 0.37

   (d) not enough information.

   (e) none of these

   $P(E^c) = 1 - P(E) = 1 - 0.37 = 0.63$

3. [5pts] Twenty athletes are to line up for a photo. There are 11 football players, 3 runners and 6 swimmers. How many ways can they line up if athletes of the same sport must be kept together?

   (a) 11!3!6!

   b (b) 3!11!3!6!

   (c) 20!

   (d) $\frac{20!}{11!3!6!}$

   (e) none of these

   T_1: arrange 3 groups of athletes : $N_1 = 3!$
   T_2 arrange 11 football pl. : $N_2 = 11!$
   T_3 — " — 3 runners $N_3 = 3!$
   T_4 — " — 6 swimmers $N_4 = 6!$
   By multiplication principle
   $N = N_1 \cdot N_2 \cdot N_3 \cdot N_4$

4. [5pts]How many distinguishable arrangements of the letters in the word *ABRACADABRA* are possible?

   (a) 39, 961, 800

   (b) 332, 640

   (c) 166, 320

   d (d) 83, 160

   (e) none of these

5	A A A A A
2	B B
2	R R
1	C
1	D

   $\frac{11!}{5! \cdot 2! \cdot 2!}$

5. [5pts] A certain restaurant has a choice of 10 appetizers, 12 entrees and 5 desserts. If you plan to order an appetizer, an entree and a dessert, how many different three-course meals are possible?

   (a) $10 + 12 + 5$

b (b) $10 \times 12 \times 5$

   (c) 10!12!5!

   (d) $P(27, 3)$

   (e) none of these

By Multipl principle

T_1 choose 1 app. from 10 | $N_1 = 10$

T_2 choose 1 entree from 12 | $N_2 = 12$

T_3 choose 1 dessert from 5 | $N_3 = 5$

6. [5pts] The Venn diagram below shows the *number* of elements in each space. Use this diagram to find $n[\mathbf{A} \cap (\mathbf{B}^c \cup \mathbf{C})]$.

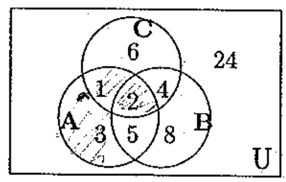
   (a) 9

   (b) 30

c (c) 6

   (d) 4

   (e) none of these



$B^c \to 1, 3, 6, 24$

$C \to 1, 6, 2, 4$

$B^c \cup C \to 1, 2, 3, 4, 6$

$A \to 1, 2, 3, 5$

$A \cap (B^c \cup C) \to 1, 2, 3$

$n(A \cap (B^c \cup C)) = 1 + 2 + 3$

7. [5pts] An experiment has a sample space of $S = \{s_1, s_2, s_3, s_4, s_5\}$ with a probability distribution as shown in the table:

Simple Event	$\{s_1\}$	$\{s_2\}$	$\{s_3\}$	$\{s_4\}$	$\{s_5\}$
Probability	0.15	0.1	0.1	p_4	p_5

If $E = \{s_2, s_4, s_5\}$, $F = \{s_1, s_3, s_5\}$ and $P(E \cap F) = 0.4$, what is p_4?

   (a) 0.2

   (b) 0.35

c (c) 0.25

   (d) 0.4

   (e) none of these

$E \cap F = \{s_5\} \Rightarrow p_5 = P(\{s_5\}) = 0.4$

$0.15 + 0.1 + 0.1 + p_4 + 0.4 = 1$

$p_4 = 0.25$

8. [5pts] Let E and F be subsets of sample space S with $P(E) = 0.44$, $P(F) = 0.49$ and $P(E \cap F) = 0.12$. Find $P((E \cup F)^c)$.

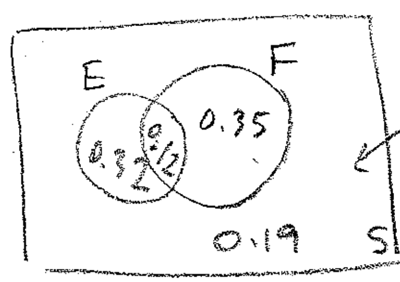
   (a) 0.2856

   (b) 0.88

   (c) 0.07

d (d) 0.19

   (e) none of these



$P((E \cup F)^c)$

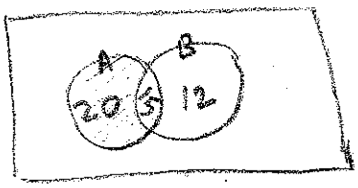
Part II - Work Out Problems You must show all appropriate work for all problems in this section to receive full credit. If you do the work in your head, you must turn in your head to get full credit. Include any intermediate steps and programs/functions you use on your calculator.

9. [6pts] If $n(A) = 35, n(A \cup B) = 47$, and $n(B) = 27$ then compute

(a) $n(A \cap B) =$ 15

$$n(A \cap B) = n(A) + n(B) - n(A \cup B) =$$
$$= 35 + 27 - 47 =$$

(b) $n(A \cap B^c) =$ 20



10. [8pts] An economy consists of three sectors: gathering (G), fishing (F), and crafting (C). The input-output matrix for this economy is given by

$$A = \begin{array}{c} \\ G \\ F \\ C \end{array} \begin{array}{c} \begin{array}{ccc} G & F & C \end{array} \\ \begin{bmatrix} 0.2 & 0.4 & 0.1 \\ 0.1 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.1 \end{bmatrix} \end{array}.$$

Find the value of each of the goods consumed in the **internal** process of production to satisfy an external demand for 474 units of gathered goods, 948 units of fishing, and 474 units of crafted products.

2pt calculator

1pt $D = \begin{bmatrix} 474 \\ 948 \\ 474 \end{bmatrix}$ $X = (I - A)^{-1} D = \begin{bmatrix} 1600 \\ 1700 \\ 1260 \end{bmatrix}$ 1pt

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

1pt

$AX = X - D = \begin{bmatrix} 1126 \\ 752 \\ 786 \end{bmatrix}$ 1pt

1pt 1pt

1126 units of gathering goods
752 units of fishing
474 units of crafted products

11. [8pts] Formulate but **DO NOT SOLVE** the following exercise as a linear programming problem. Be sure to clearly define the variables.

A contractor builds two type of homes. Each type A home requires \$12,000 in capital and 150 labor-days to build and is sold for a profit of \$29,000. Each type B home requires \$28,000 in capital and 200 labor-days to build and is sold for a profit of \$45,000. The contractor has \$2,925,000 in capital and 24,000 labor-days available for the job. If the contractor is under the additional constraint from a city ordinance that the number of type B houses cannot exceed the number of type A houses, then how many houses of each type should the contractor build to realize the greatest profit?

	Type A	Type B	
#homes	x	y	
Capital	$12000x$	$28000y$	$\le 2,925,000$
Labor-days	$150x$	$200y$	$\le 24,000$
Profit	$29000x$	$45000y$	$\to$ max

$y \le x$

x # of type A houses

y # of type B houses

Maximize $P = 29000x + 45000y$

Subject to: $12000x + 28000y \le 2,925,000$

$150x + 200y \le 24,000$

$y \le x$

$y \ge 0$

12. [6pts] An experiment consists of selecting a letter at random from the letters in the word *ABRACADABRA* and observing the outcomes.

(a) What is an appropriate sample space for this experiment?

$S = \{A, B, R, C, D\}$

(b) Describe the event "the letter selected is NOT a vowel".

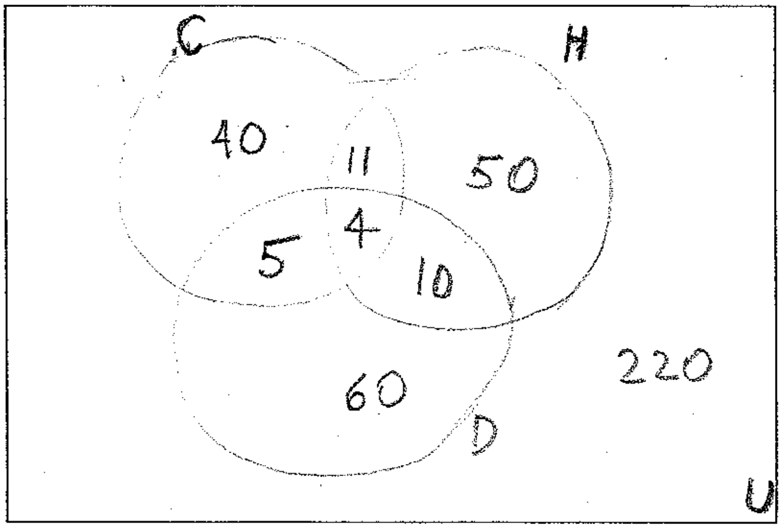
$\{B, R, C, D\}$

13. [10pts] A group of 400 college students was asked about the types of pets they have, specifically, whether they have a cat, dog, or hamster. The survey revealed the following:

- 120 students have not *cat* but have *hamster* or *dog*; $n(C^c \cap (H \cup D)) = 120$
- 61 have *hamster* but not *dog*; $n(H \cap D^c) = 61$
- 15 have *cat* and *hamster*; $n(C \cap H) = 15$
- 14 have *hamster* and *dog*; $n(H \cap D) = 14$
- 9 have *cat* and *dog*; $n(C \cap D) = 9$
- 5 have *cat* and *dog* but not *hamster*; $n((C \cap D) \cap H^c) = 5$
- 40 have only *cat*. $n(C \cap (H \cup D)^c) = 40$

Let C = *cat*, H = *hamster* and D = *dog*;

Fill in a Venn diagram illustrating the above information.



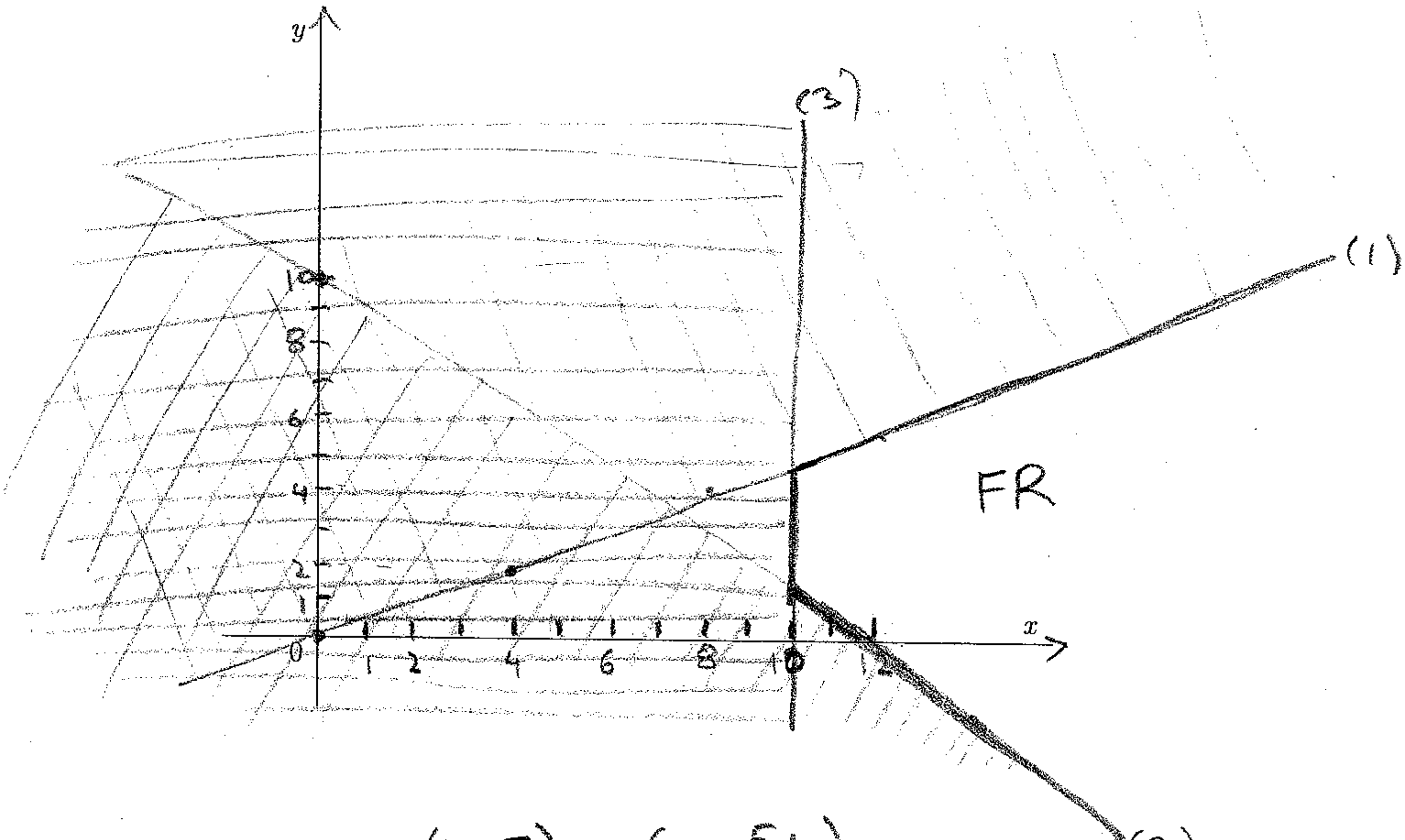
14. [6pts] In solving a linear programming problem, you obtain a bounded feasible region with the corner points $(1,2), (5,1), (2,5), (6,4)$. Find the minimum value of the objective function $P = 2x + y$ on this feasible region and specify where the minimum value occurs. (Fill in the chart provided).

Corner	P
(1,2)	4
(5,1)	11
(2,5)	9
(6,4)	16

$\min P = 4$ at $(1,2)$

15. [8pts] Graph the feasible region. You may do reverse shading or regular shading. Please indicate where the feasible region is located with FR. Label the lines in your picture. Also list all corner points.

$$\begin{aligned} x - 2y &\geq 0 \quad &(1)\\ 10x + 12y &\geq 120 \quad &(2)\\ x &\geq 10 \quad &(3) \end{aligned}$$



Corner points: (10, 5), (10, 5/3)

(1) ∩ (3) → $\begin{cases} x = 10 \\ x - 2y = 0 \end{cases}$ → $10 - 2y = 0$, $y = 5$

(2) ∩ (3) → $\begin{cases} x = 10 \\ 10x + 12y = 120 \end{cases}$

$100 + 12y = 120$

$y = \frac{20}{12} = \frac{5}{3}$

Part III True/False

16. [2pts each] *CLEARLY circle TRUE or FALSE for each statement.*

TRUE	FALSE	$2 \subset \{1, 2, 3, 4, 12\}$ $\in$
TRUE	FALSE	Let $A = \{a, b, c, d\}$ and $B = \{a, c, e\}$. Then $A \cap B = \{a, c\}$.
TRUE	FALSE	If $A \cap B = \emptyset$ then $B = \emptyset$.
TRUE	FALSE	Let $A = \{a, b, c, d\}$ and $B = \{a, c, e\}$. Then $n(A \cup B) = 2$.