

Lecture 5
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This week's topic is Hamiltonian graphs in which we want to hit every vertex once without repeating a vertex, but it doesn't matter what edges we use. Indeed, in most graphs we won't use all the edges. As is in the reading, Alexander Hamilton started the study of Hamiltonian graphs by creating a traveling salesmen game.

Since we are skipping Exercise 3.7, I include the proof of Dirac's Theorem here so that you can use it in your solutions to the assigned Problems.

Proof: Suppose that there are n vertices. After adding edges, we may assume that there is a path of the form (v_1, v_2, \dots, v_n) , where v_1 and v_n are not adjacent. I claim that the degree condition implies that, if we look at all the vertices adjacent to v_n , there must be one, say v_k , whose successor is adjacent to v_1 . For v_1 is connected to at least $n/2$ vertices and v_n is connected to at least $n/2$ vertices but v_1 is not connected to v_n . Therefore they both must be connected to at least one vertex, v_k . Then a Hamiltonian cycle is $(v_1, \dots, v_k, v_n, v_{n-1}, \dots, v_{k+1}, v_1)$.

Problem 3.7 hint: Change the problem into one for a graph constructed from the information given.

On Problem 3.8 be sure to prove your characterization, i.e., a necessary and sufficient condition that the graph have a path that is both Eulerian and Hamiltonian.