CHAPTER 17 – INFORMATION SCIENCE

Binary and decimal numbers – a short review:

For decimal numbers we have 10 digits available (0, 1, 2, 3, … 9) with place values as powers of ten.

______	______	______	______	______
10,000	1000	100	10	1
10^4	10^3	10^2	10^1	10^0

4731 =

For binary numbers we have 2 digits available (0 and 1) with place values as powers of two.

____	____	____	____	____	____	____
64	32	16	8	4	2	1
2^6	2^5	2^4	2^3	2^2	2^1	2^0

Express the following binary numbers as decimal numbers:

10111_{Two} =

11101010_{Two} =

Express the following decimal numbers as binary numbers:

57 =

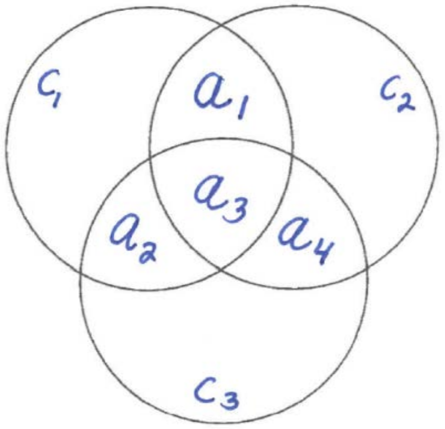
71 =

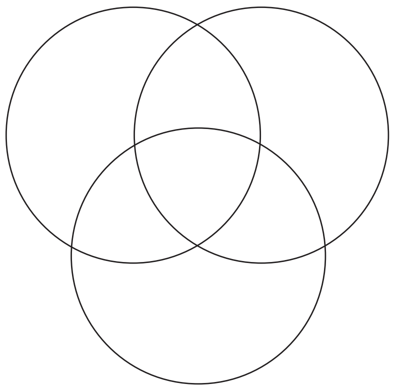
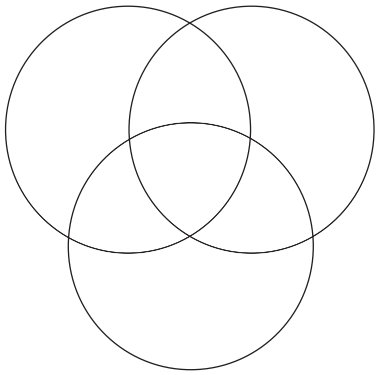
Using binary digits (bits), we can encode up to 2^n pieces of information using *n* bits. So how many bits will we need to code 16 pieces of information?

Some numbers have multiple check digits. In some cases, the check digits are arranged so that certain sums have even parity. These are called ***parity-check sums*** where the parity of a number refers to whether a number is even or odd. Even numbers have ***even parity*** and odd numbers have ***odd parity***.

A set of words composed of 0's and 1's that has a message and parity check sums appended to the message is called a ***binary linear code***. The resulting strings are called ***code words***.

For this type of parity-check sum with 4 bit codes, $a_1a_2a_3a_4$, and 3 binary check digits, $c_1c_2c_3$, we can use a Venn diagram to help find the check digits or find errors when there is only one error.



a) What is the appropriate code word (code plus check digits) for the code 1001?

b) Fix the error in the code 1101101 if it is known only one digit has an error.

The process of determining the message you were sent is called ***decoding***. If you are sent a message *x* and receive the message as *y*, how can it be decoded?

The ***distance between two strings*** of equal length is the number of positions in which the strings differ.

(a) 10111 and 11101 distance of _______

(b) 110101 and 101010 distance of ______

The ***nearest neighbor decoding method*** decodes a message as the code word that agrees with the message in the most positions *provided there is only one such message*.

How good a code is at detecting and correcting errors is determined by the weight of the code. The ***weight of a binary code*** is the minimum number of 1's that occur among all non-zero code words of that code.

Consider a code of weight *t*,

- The code can *detect* $t - 1$ or fewer errors.
- The code can *correct* half as many as it can detect (rounded down).
  - $\frac{t-1}{2}$ or fewer errors if *t* is odd.
  - $\frac{t-2}{2}$ or fewer errors if *t* is even.

Consider the code C = {0000000, 0011111, 1111000, 1111111}

(a) What is the weight of the code?

(b) How many errors can this code detect?

(c) How many errors can this code correct?

(d) Decode the message received as 0001011 using nearest neighbor.

A ***compression algorithm*** converts data from an easy-to-use format to one that is more compact. jpg photo files use data compression as do most video and audio files.

Delta function encoding uses the beginning value and the differences in one value to the next to encode the data.

Compress the data below using delta function encoding and determine how much the data is compressed.

1361 1357 1349 1350 1351 1351

Decompress the following data that was coded using delta function encoding:

1027 3 -2 5 6 0 -3

Binary codes can also be compressed by assigning short codes to characters that occur frequently and longer codes to characters that occur rarely.

We have 5 symbols, A, B, C, D, and E. If we give all the symbols a code of the same length, we would need 3 binary digits (000 to 101). So a string of 6 symbols would be 6 x 3 = 18 characters long. Can we devise a different binary code if we knew how often each character occurred?

Use ***Huffman coding*** is a way to assign shorter code words to those characters that occur more often.

Step 1	Arrange these letters from least to most likely.
Step 2	Add the probabilities of the two least likely characters and combine them. Keep the letter with the smaller probability on the left. Arrange the new list from least to most likely.
Step 3	Repeat Step 2 until all the letters have been combined into one group with probability of 1.
Step 4	To assign a binary code to each letter, display the information in a Huffman tree by undoing the process from Steps 2 and 3. Always keep the smaller probability on the left and assign a 0 to that branch. Assign a 1 to the branch with the higher probability.
Step 5	The 0’s and 1’s for each path determine the code word for that letter. Read from the top of the chart down to the letter.

A	B	C	D	E
0.17	0.25	0.34	0.06	0.18

A	B	C	D	E

Decipher a message that was encoded using this Huffman code:

101101000101101110110100001111

The process of disguising data is called encryption. Cryptology is the study of making and breaking secret codes.

A ***Caesar cipher*** shifts the letters of the alphabet by fixed amount.

EXAMPLE

Create a Caesar cipher that shifts the alphabet by 8 letters and use it to encrypt the message AGGIE.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

The message TAIPK was created with a Caesar cipher with a shift of 12. What is the original message?

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

A ***decimation cipher*** multiplies the position of each letter by a fixed number k (called the ***key***) and then uses modular arithmetic. To use a decimation cipher,

1. Assign the letters A – Z to the numbers 0 – 25.
2. Choose a value for the key, k, that is an odd integer from 3 to 25 but not 13 (why not?)
3. Multiply the value of each letter (i) by the key (k) and find the remainder when divided by 26.

4. To decrypt a message, the encrypted value x needs to be multiplied by the decryption letter j and then the remainder mod 26 is the original letter.

There is a chart on page 618 of your book showing the decryption key, j, for all possible values for the encryption key, k. Pairs of encryption and decryption values include (3,9), (5,21), (7,15), (11,19), (17,23), and (25, 25). The decryption key is chosen so that $kj \equiv 1 \bmod 26$. Which also means that $(kj) \bmod 26 = 1$. You do not need to memorize this chart.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Use a decimation cipher with a key of 7 to encrypt AGGIE

	A	G	G	I	E
Position					
(Pos)*(e.key)					
Mod 26					
Code					

The message below was encrypted with a key of 9.
The decryption key is 3. Decode the message.

	L	W	V	U	B	A	I	G
Position								
(Pos)*(d.key)								
Mod 26								
Message								

A ***Vigenère cipher*** uses a ***key word*** to encode the characters.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

Use a ***Vigenère cipher*** with a key word of WIN to encode the message

	A	G	G	I	E
Position					
Key Word					
Sum					
Mod 26					
Code					

A ***Vigenère cipher*** with a key word of YES was used to encode the message below. Decode it. You will need to subtract rather than add.

	Q	X	M	B	C
Position					
Key Word					
Diff.					
Mod 26					
Code					

To increase security, binary strings can be added together. If the result is even, enter 0. If the result is odd, enter 1. In other words, take the sum of the digits mod 2. *Note*: This is not the same as binary addition.

Add the binary strings:

(a)
```
  10110
+ 00111
```

(b)
```
  10110
+ 10110
```

SAMPLE EXAM QUESTIONS FROM CHAPTER 17

1. Convert the binary number 10011 to a decimal number.

(A) 3 (B) 19

(C) 16 (D) 12

2. What is the distance between received words 1110101 and 1010111?

(A) 1 (B) 2 (C) 3 (D) 4

(E) more than 4

3. Add the binary strings 1101101 and 1110101. How many 1s digits are in the sum?

(A) 1 (B) 2 (C) 3 (D) 4

(E) more than 4

4. Use delta encoding to compress the data

1724 1721 1721 1715 1739.

By how many characters is the data compressed?

(A) 9 (B) 10

(C) 11 (D) 13

9. Use the Venn diagram method to find the code word for the code 1100.

5. Use the Venn diagram method to decode the received word 1011101 assuming there was only 1 error.

(A) 1001

(B) 0100

(C) 1101

(D) 1011

(E) None of these

<u>Questions 6 and 7 use the code {1110, 1011, 1101, 0110, 0101, 0011}.</u>

6. What is the weight of this code?

(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. Which one of the following is a true statement about this code?

(A) This code can detect and correct two errors

(B) This code can detect two errors and correct 1 error

(C) This code can detect and correct one error.

(D) This code can detect one error and correct 0 errors

(E) None of these

8. Given binary codes A $\rightarrow$ 0, C $\rightarrow$ 10, I $\rightarrow$ 110, S $\rightarrow$ 1110, B $\rightarrow$11110.

(a) Encode the message SABAAC

(b) Decode the message 11011101000110

10. Use a Huffman code to assign binary codes to the directions that occur with the probabilities given below.

Up	Down	Left	Right
0.32	0.16	0.03	0.49

11. Use a Caesar cipher with a shift of 6 to encode the word BINARY.

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

12. Use a decimation cipher with key 17 to encode the word CABLE.

	C	A	B	L	E

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z

13. Use the Vigenere cipher with the key word GOLD was used to encode LCZWHOWO. Decode the message.

	L	C	Z	W	H	O	W	O