Bernoulli equations.

Sometimes it is possible to solve a nonlinear differential equation by making a change of the dependent variables that converts it into a linear equation. The most important such equation is of the form

$$y' + p(t)y = q(t)y^{\alpha}, \quad \alpha \neq 0, 1 \tag{1}$$

and it is called Bernoulli equation after Jakob Bernoulli who found the appropriate change (note that for $\alpha = 0, 1$ such equation is already linear).

Indeed, let

$$v(t) = y(t)^{1-\alpha} \tag{2}$$

Then by the chain rule:

$$v' = (1-\alpha)y^{-\alpha}y' \tag{3}$$

Dividing (1) by y^{α} we get

$$y^{-\alpha}y' + p(t)y^{1-\alpha} = q(t), \tag{4}$$

Then, substituting v and v' instead of y and y' from relations (2) and (3) into (4). we get

$$\frac{1}{1-\alpha}v' + p(t)v = q(t),$$

which is a linear nonhomogeneous equation and can be solved by the method of integrating factor of section 2.1.

After finding $v(t)$ return to the original $y(t)$ via relation (2).