13.4: Motion in Space: Velocity and Acceleration (very shortly as most of this material was in principle discussed in the previous two sections)

Suppose a particle moves through space so that its position vector at time t is $\mathbf{r}(t)$. Assume that $\mathbf{r}(t)$ is twice differentiable, i.e. each component has all derivatives up to order 2 at every point.

- The *velocity* of the particle at t is $\mathbf{v}(t) := \mathbf{r}'(t)$.
- The *speed* of the particle at t is the magnitude of the velocity, $|\mathbf{v}(t)| = |\mathbf{r}'(t)|$.
- The *acceleration* pf the particle at t is $\mathbf{a}(t) = \mathbf{v}'(t) = \mathbf{r}''(t)$.

EXAMPLE 1. *Find the velocity, speed, and acceleration of a particle with the given position function*

$$\mathbf{r}(t) = -\sqrt{2}t\,\mathbf{i} + e^t\,\mathbf{j} + e^{-t}\,\mathbf{k}$$

EXAMPLE 2. *Find the position vector of a particle that has the given acceleration and the specified initial velocity and position.*

$$\mathbf{a}(t) = (t^2 - t)\mathbf{i} + \cos 3t\mathbf{j} + e^{-2t}\mathbf{k},\ \mathbf{v}(0) = 2\mathbf{i} - 3\mathbf{j},\ \mathbf{r}(0) = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$$

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