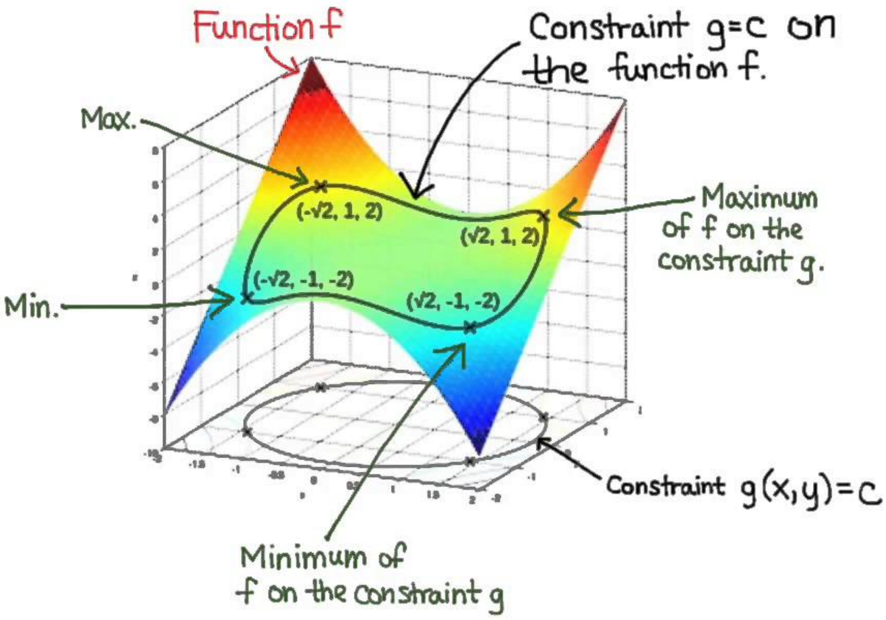
14.8: Lagrange Multipliers

PROBLEM: *Maximize/minimize a general function* $z = f(x, y)$ *subject to a constraint (or side condition) of the form* $g(x, y) = c$.



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METHOD OF LAGRANGE MULTIPLIERS: *To Maximize/minimize a general function* $z = f(x, y)$ *subject to a constraint of the form* $g(x, y) = c$ *(assuming that these extreme values exist):*

1. *Find all values* x, y *and* λ *(a Lagrange multiplier) s.t.*

$$\nabla f(x, y) = \lambda \nabla g(x, y)$$

and

$$g(x, y) = c$$

2. *Evaluate* f *at all points* (x, y) *that arise from the previous step. The largest of these values is the* $\max f$*; the smallest is the* $\min f$.

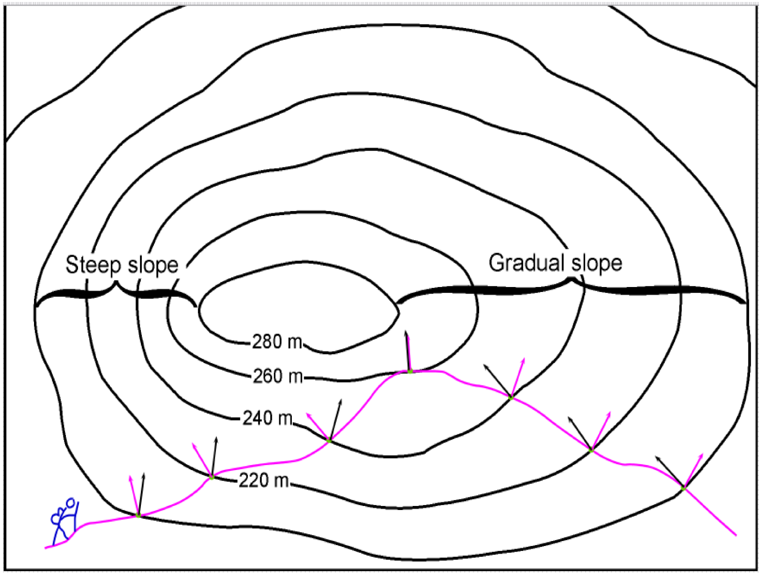
Rewrite the system

$$\nabla f(x, y) = \lambda \nabla g(x, y)$$
$$g(x, y) = c$$

in component form:.

EXPLANATION via properties of gradient

Visual explanation



https://math.stackexchange.com/questions/686538/how-to-explain-lagrange-multipliers-to- a - l a y - a u d i e n c e/686655

EXAMPLE 1. *Use Lagrange multipliers to find the maximum and minimum values of* $f(x, y) = x^2 + y^2$ *subject to* $x^4 + y^4 = 1$.