End of the class on Wednesday, June 27 2012

We end up with the question how to find the Laplace transform of $(t-1)u_3(t)$ and of $(t^2-t)u_5(t)$. We want to use the translation in t property:

$$\mathcal{L}\{u_c(t)f(t-c)\} = e^{-cs}F(s), \tag{1}$$

where $F(s) = \mathcal{L}\{f(t)\}$.

1. For $(t-1)u_3(t)$ we want to find $f(t)$ such that $f(t-3) = t-1$. For this

$$f(t) = f((t+3)-3) = (t+3)-1 = t+2$$

In other words, to find $f(t)$ from the knowledge that $f(t-3) = t-1$ we replace t by $t+3$ in $t-1$. Further, $F(s) = \mathcal{L}\{t+2\} = \frac{1}{s^2} + \frac{2}{s} = \frac{2s+1}{s^2}$. Therefore by (1) (with $c = 3$, $F(s) = \frac{2s+1}{s^2}$) we have

$$\mathcal{L}\{u_3(t)(t-1)\} = e^{-3s}\frac{2s+1}{s^2}.$$

2. For $(t^2-t)u_5(t)$ we want to find $f(t)$ such that $f(t-5) = t^2-t$. For this, as in the previous case, it is enough to replace t by $t+5$ in t^2-t:

$$f(t) = (t+5)^2 - (t+5) = t^2 + 10t + 25 - t - 5 = t^2 + 9t + 20 \Rightarrow$$

$$F(s) = \frac{2}{s^3} + \frac{9}{s^2} + \frac{20}{s} = \frac{20s^2+9s+2}{s^3}$$

Finally, again by (1) (with $c = 5$ and $F(s) = \frac{20s^2+9s+2}{s^3}$) we get

$$\mathcal{L}\{u_5(t)(t^2-t)\} = e^{-5s}\frac{20s^2+9s+2}{s^3}.$$