Topics for Exam 1, MATH309-Spring 2013

1. Sections 1.1-1.2: To know the notion of consistent and inconsistent linear systems; how to transform the system to a row echelon form and then to solve it by the back substitution; how to transform the system to a reduced row echelon form and then to solve it, to know the notion of leading and free variables with respect to the row echelon form; how to decide whether the system is consistent and about the number of solutions of it from the row echelon form; to know the notion of underdetermined and overdetermined linear system, and to know the properties of homogeneous systems (as described in page 20)

2. Sections 1.3,1.4,1.5: To know the operation on matrices: multiplication by a scalar, the matrix addition, the matrix multiplication and their properties as in Theorem 1.4.1 pages 44-45, in particular, to understand that the matrix multiplication is not commutative; to know the notion of the transpose of the matrix and its properties as at the end of page 52; the notion of symmetric and skew-symmetric matrices; the notion of nonsingular (invertible) matrix and various its characterization as described in Theorem 1.5.2 on page 61, Corollary 1.5.3 on page 62; to know the formula for the inverse of the product of two non-singular matrices (as described in Theorem 1.4.2, page 52), to know how to find the inverse of 2×2 matrix using the formula of exercise 12 on page 56 or , equivalently, of example 1 on page 99, and the inverse of $n \times n$ matrix using Jordan-Gauss reduction as described in section 1.5.

3. Sections 2.1-2.3 and my lecture notes/worksheets posted on

   http://www.math.tamu.edu/~zelenko/S13supp.html

   To know the notion of the minor and of the cofactor of an entry of a matrix as on page 87 and how to calculate the determinant inductively via the expansion along a row or a column. To know the notion of a permutation, an inversion of a permutation, the signature and the parity of a permutation, the expansion of the determinant via permutations and how to determine the sign of any given term in this expansion both by counting the number of inversions of the corresponding permutation and by counting the parity of a number of transposition needed to transform the corresponding permutation to the the permutation with the natural order. To know how to use the row/column operations in calculation of the determinant, i.e. what happens with the determinant if we interchange two rows or two columns, if we apply a row/ a column by a scalar, if we add to one row/column another row/column multiplied by a scalar (i.e. items I, II, III of the Summary on page 94). To know how to determine whether a matrix is invertible via the determinant (Theorem 2.2.2, page 94). To know the property that the determinant of the product of matrices is equal to the product of their determinants (Theorem 2.2.3, page 96). To know how to find the inverse of the matrix using adjoint (the boxed formula on page 99). To know how to solve a linear equation using Cramer's rule (Theorem 2.3.1, page 100).

4. Sections 3.1-3.2

   To know how to check whether a set is a vector space (the definition of a vector space with all axioms will be included in the text of the exam), to know examples of vector spaces such as $\mathbb{R}^n$, the space of all matrices of the given size, the space of polynomials P_n of degree less than n, the space $C[a, b]$ of continuous functions on the interval $[a, b]$; to know how to prove some simple properties of vector spaces using axioms (Theorem 3.1.1 on page 115); to know how to check whether a given subset of a vector space is a subspace of it; to know how to find the null space of a matrix; to know the notions of a linear combination, of the span of the given vectors, of the spanning set of a vector space; to know how to check whether a vector can be represented as a linear combination of the

given vectors and, if yes, how to find this representation; to know how to determine whether a given set is a spanning set of a given vector space.

It is recommended to review all problems in homework assignments 1-4 and the examples given during the class on the topics listed above. Also it will be very useful as a part of your preparations to the test to practice the following problems from the Chapter tests:

Chapter 1 Test A p. 82: problems 1-12 and problem 15;

Chapter 1 Test B, pp. 82-83: problems 1-4, 6-9;

Chapter 2 Test A, p. 108: problems 1-10;

Chapter 2 Test B, pp. 108-109: 1 (a) (b), 2, 4, 5, 6, 7, 8, 9, 10;

Chapter 3 Test A, p 164: problems 1-5;

Chapter 3 Test B, p. 164: problem 2, 5, 6(a), 8(b), (c) (the first question there), (d) (the first question there).