

Topics for Exam 3, MATH309-Spring 2013

Please read carefully all items below. Then make the review topic by topic. Note that the exam will cover most of the topics below. Do not put yourself in the situation that you come to the test without reviewing some of the topics. If you struggle with a topic, do not hesitate to come to my office hours or send me an email with your concerns. In addition to the regular office hours: M 2-3pm, W 10-11 a.m, F 11-12 a.m., Milner 324., I will give special office hours for test 2 on Tuesday, April 23, 10 a.m-1 p.m. and Thursday, April 25 11 a.m.-3 p.m.

Here are the topics:

Part 1. Orthogonality: Chapter 5 of the book (sections 5.1, 5.4, 5.5)

1. Section 5.1. *Scalar product in \mathbb{R}^n* . To know how to find the scalar product between two vectors in \mathbb{R}^n , the length of the vector in \mathbb{R}^n , the distance between two points in \mathbb{R}^n , the angle between two vectors in \mathbb{R}^n , the scalar and vector projection of one vector to another, the distance from a point to a line, the equation of hyperplane in \mathbb{R}^n and the distance from a point to a hyperplane; to know how to determine whether given vectors are orthogonal; to know the Cauchy-Schwarz-Bunyakovsky in \mathbb{R}^n inequality and how to use it.
2. Section 5.4 *Inner product space*. To know the definition of an inner product in a vector space (Definition on page 232). To be aware about various inner products: the inner product in \mathbb{R}^n with given weight vector (as in formula (1) of page 232), the inner product in the space of matrices (as in formula (2) of page 232), the inner product in the space of continuous functions as in formula (3) of page 232 and the inner product in the same space corresponding to a weight function as in formula (4) of page 233, the inner products in the space of polynomials, given by the formula (5) of page 233. To know how to find the same quantities as in the item 1) with respect to a given inner product in a given vector space V , i.e. to find the length of the vector in V , the distance between two points in V , the angle between two vectors, the scalar and vector projection of one vector to another; to know how to determine whether given vectors are orthogonal and the Pythagoras Theorem; to know the Cauchy-Schwarz-Bunyakovsky inequality and how to use it.
3. Section 5.5 *Orthonormal sets*. To know to determine whether a given set of vectors is orthogonal or orthonormal. To know how to find the coordinates of a given vector with respect to an orthonormal basis (Theorem 5.5.2 page 243) and to know how to find the inner product and the length of the vector via its coordinates with respect to the orthonormal basis (Corollaries 5.5.3 and 5.5.4 in the same page). To be aware that the set $(\frac{1}{\sqrt{2}}, \cos x, \sin x, \cos 2x, \sin 2x, \dots, \cos nx, \sin nx, \dots)$ is an orthonormal set in $C[-\pi, \pi]$ with the inner product $\langle f, g \rangle = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x)g(x) dx$. To know how given a point v in an inner product space to find the closest point to this point in a given subspace if an orthonormal basis is given in this subspace (as in the problems 29 and 30 of the homework assignment #10, this closest point is called the best least square approximation)

Part 2. PDE's :Classification, Separation of variables, Fourier series, half range Fourier series, Applications to the heat equation and Laplace equation on a rectangle.

1. To know to classify second order linear PDE's of two independent variables as hyperbolic, parabolic, or elliptic (lecture 1, class of April 10, pages 5-8);

See the next page!

2. To know to determine whether a PDE is separable and if yes to be able to write down the resulting ODE's. (lecture 3, class of April 15);
3. To know how to find the Fourier series of a periodic function. To know the Dirichlet condition and to what function the Fourier series of a function satisfying Dirichlet condition converges (especially what happens at the points of discontinuity of the original function). To know the Parseval identity for the coefficients of the Fourier series (lecture 4, class of April 17)
4. To know how to find the half range Fourier sine and cosine series of a function, to know their relation to odd and even functions, to know to to what function the half range Fourier series converges (lecture 5, class of April 19)
5. To know how to apply the method of separation of variables and half-range Fourier series to the heat equation with two types of boundary conditions: both ends are kept with zero temperature or both ends are insulated (lecture 6 of class of April 22, pages 1-9).
6. To know how to apply the method of separation of variables and half-range Fourier sine series to the Laplace equation in a rectangle with Dirichlet boundary conditions (lecture 6 of class of April 22, pages 10-15).

It is recommended to review all problems in homework assignments 9-11 and all examples given during the class on the topics listed above (for the PDE part review lecture notes 1-6 posted in the course webpage). Also it will be very useful as a part of your preparations to the test to practice the following problems from the Chapter 5 test B, pp 280-281: problems 1, 2, 3, 8, 11, 12: